THE

PHYSICAL SOCIETY

OF

LONDON.

PROCEEDINGS.

VOLUME XXXI.—PART V.

AUGUST 15, 1919.

Price to Non-Fellows, **4s.** *net, post free* **4/3.**

Annual Subscription, **20/-** *post free, payable in advance*

Published Bi-Monthly from December to August.

LONDON:
FLEETWAY PRESS, LTD.,
1, 2 AND 3, SALISBURY COURT, FLEET STREET.

1919.

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A New Colour Transparency Process for Illustrating Scientific Lectures. *By* A. E. Bawtree, *F.R.P.S.*

Methods of illustrating scientific lectures may be divided into three classes : Blackboard drawings and wall diagrams, experiments and optical lantern slides. The first is unsatisfactory, especially before large audiences ; the second entails much preparation and some risk of failure, and is limited in its usefulness ; the third method is of far the widest application. For astronomical, microscopic and archæological subjects and for radiographs and other purely photographic records, monochrome photographic slides leave nothing to be desired, but for many purposes colour is necessary. Owing to the large scale of magnification upon the screen, only the crudest washes can be obtained by hand painting. Processes of natural colour photography are excellent when there is no very fine line work which would be destroyed by the grain of the image ; but some subjects require definition of microscopic fineness combined with brilliant colouring. It was in order to illustrate a Paper before the Engineering Section of the British Association upon a method of bank note engraving that I devised the process illustrated.

The image is produced in a thin colloid film upon bare glass. Considerable experimenting led to the selection of a range of dyes and mordants by which practically any shade of the most brilliant colouring could be obtained. By suitable insulating films, images in any number of colours can be superimposed and accurately registered with one another. Thus, diagrammatic slides can be prepared in various colours. The passage of a beam of white light through a prism can be shown spreading out into bands of colour, instead of merely initialled lines. Coloured mosaics can be placed in a diffusing lantern to show the preparation of additive colours—*e.g.*, red and green producing yellow, more convincingly and brilliantly than with the Maxwell disc.

The process has other applications than the preparation of lantern slides. If a screen of lines ruled in alternate red and green of, say, 250 lines per inch, have projected upon it by means of a lens the image of a screen of similar ruling, but in lines of alternate black and white, the former screen will appear to change colour with a movement of the latter image of only 1/250 of an inch. Thus, this small motion can be rendered strongly visible to the largest audience. In this

manner compressive or tensile strain in a rod or the expansion of a rod with heat can be shown. By means of a balance beam fitted with a scale pan at one end and a bell crank at the other end, very small pressures—say 10 milligrams—can be made to act upon a heavy body; for example, a hollow leaden ball of 10 kilos weight just floating in water. By such an apparatus, combined with the two screens, the laws of fluid friction and of motion can be beautifully illustrated. In moving one inch, the ball will cause the screen to change colour 250 times, and these colour changes can be plotted against time in a variety of experiments. Numerous other applications of the method of preparing any form of colour transparency will doubtless suggest themselves to lecturers, and the process should place a useful additional means of demonstration in their hands.

XVII. *Absolute Scales of Pressure and Temperature.* By F. J. W. Whipple, *M.A.*

Received March 28, 1919.

It has been suggested to me that it would be fitting to call the attention of members of the Physical Society to the adoption by meteorologists of new scales of pressure and temperature, and to canvass the advantages of bringing such into general use.

The fundamental reason for adopting an absolute dynamical unit for pressure lies in the fact that the variation of gravity from place to place on the surface of the globe is quite appreciable. In practical meteorology pressures are required accurate to one part in 10,000, so that the allowance for this variation, which is of the order $\frac{1}{2}$ per cent., cannot legitimately be ignored. For many years it was customary to publish values of pressure in terms of the height of the barometer, making no allowance for this variation. In the meteorological charts, isobars were drawn to pass, not through places where the pressure was the same, but through places where the "head" of mercury was the same. A later development (which occurred in the British service in 1912) was to "reduce" the barometric readings to latitude 45°, the unit of pressure being the pressure due to an inch of mercury in latitude 45°. Such a unit is very artificial and is also irrelevant. The only satisfactory way to avoid the difficulty is to use absolute units which can be defined simply in terms of force and area.

The fundamental unit which has actually been adopted is the bar, the pressure due to a million dynes per square centimetre. The practical unit for meteorological work is the millibar, or 1,000 dynes per square centimetre. This unit is rather more than the gravitance of 1 gramme per square centimetre; it would be exactly the gravitance of 1 gramme per square centimetre at a place where the acceleration due to gravity was 1,000 cm./sec.^{2}. It is worth noticing that the millibar would be the pressure due to 1 cm. of water at maximum density at a place where gravity had this value.

It is necessary to emphasise the fact that the bar is not the same as the standard atmosphere used hitherto by the chemist, 760 mm. of mercury at 0°C. in latitude 45°. Actually the

bar is equivalent to 750·076 mm. of mercury under those conditions.*

The millibar is now in use, not only throughout the British Meteorological Service, but also in France. Other countries are gradually adopting it. It should be mentioned, however, that a different nomenclature is advocated by Prof. McAdie, of Blue Hill Observatory, which is attached to Harvard University. He wishes the name bar to be utilised for 1 dyne/cm.², so that our millibar would become the kilobar. He bases his advocacy on the tentative practice of a few chemists, but the weight of practical experience is in favour of the larger fundamental unit.

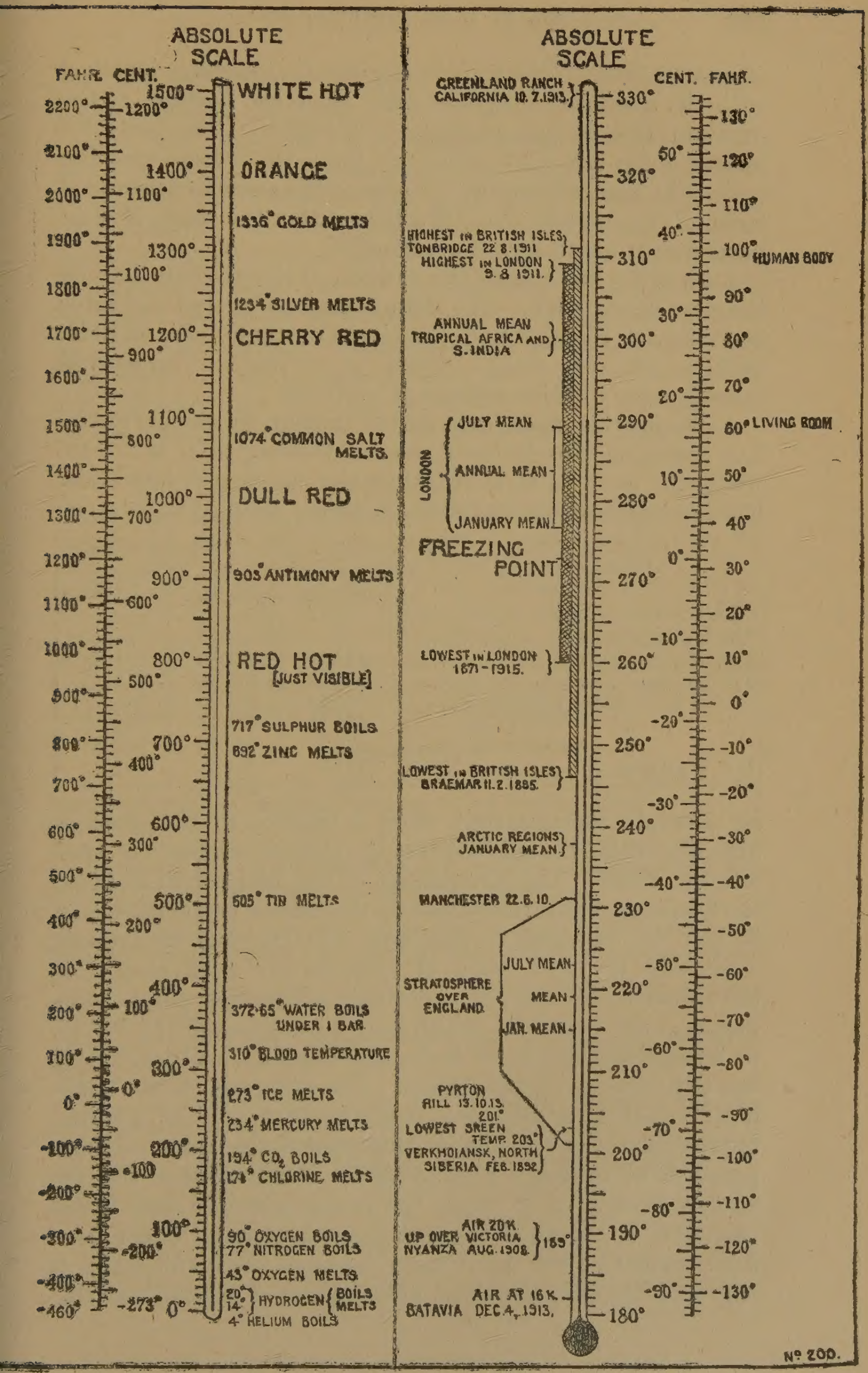
Instrument makers are now familiar with the graduation of barometers in millibars, and it is to be hoped that such instruments will shortly be introduced into all physical laboratories.

Another movement which was inaugurated about the same time as the first introduction of the millibar was that in favour of the everyday use of the absolute scale of temperature. The customary scales of temperature involve the frequent introduction of negative temperatures which are thoroughly illogical. With a scale such as the centigrade we are met with the paradox that $-272°$C. has a definite meaning, whilst $-274°$C. is unthinkable. It is hardly necessary to point out to the Physical Society that in thermodynamics it is always absolute temperature which counts. In meteorology we have to consider certain practical aspects of the question. In the first place, as may be seen in the accompanying diagram, the range of temperature with which modern meteorology, whose province is the whole atmosphere, is concerned goes from† 180a to 330a ($-130°$F. to $130°$F.), so that in many investigations negative temperatures would abound if the Fahrenheit or centigrade scales were used; and in the second place, on the theoretical side, the application of the laws of gases are so frequent that the use of the absolute scale leads to a direct economy of thought.

The temperature scale which has actually been adopted by meteorologists is derived from the centigrade by the addition

* This relation is derived from the equations $g_{45}=980·617$ cm./sec.²; density of mercury at normal freezing point of water$=13·5955$ gm./cm.² Conversion tables for pressures will be found in the Computer's Handbook of the Meteorological Office.

† Sir Napier Shaw advocates writing 180a rather than 180°A on the ground that the degree symbol ° should be reserved for angles.



TEMPERATURE SCALES IN PHYSICS AND METEOROLOGY.
(Reproduced by permission from "The Weather of the British Coasts.")

of exactly 273°. This scale is not, strictly speaking, identical with the absolute centigrade scale, since the position of the absolute zero is not precisely −273·000°C. The meteorologist's scale has been called by some the pseudo-absolute scale. Sir Napier Shaw has introduced the name tercentesimal scale, on the ground that 300a falls within the range of temperatures to which we are accustomed.

Authorities differ as to the position of the absolute zero on the centigrade scale. The determinations quoted by Kaye and Laby (" Physical and Chemical Constants," 3rd edition, 1918, p. 44) vary from −273·05 to −273·27. The general mean is given by these authors as −273·13.

Accepting this mean result for the present, we see that on the true centigrade-absolute scale normal freezing point is 273·13, whilst the same point is 273·00 on the pseudo-absolute scale. The few certificates issued by the National Physical Laboratory

—	Absolute centigrade.	Pseudo-absolute.	I.F.P.
Absolute zero	0	−0·13	0
F.P. of mercury	234·26	234·13	234·15
F.P. of water	273·13	273·00	273·00
B.P. of water under 1013·2 mb.	373·13	373·00	372·95
,, ,, 1015·1 mb.	373·18	373·05	373·00
,, ,, 1000·0 mb.	372·78	372·65	372·60
B.P. of sulphur	717·8	717·7	717·5

for absolute thermometers refer, I believe, to the pseudo-absolute scale, though there is no explicit evidence to that effect on them. The convenience of the adoption of 273a for the freezing point rather than 273·13a cannot be gainsaid, and it may be desirable to regularise the position by defining a slightly modified scale of temperature such that 0 is the absolute zero and 273 is the normal freezing point. The degree of this new scale, which may be designated the I.F.P., or integral-freezing-point scale, provisionally, will be slightly less than the centigrade degree. The difference is so small, however, that hardly any measurements will be affected appreciably. For example, throughout the range of temperature of ordinary meteorological thermometers the departure of the I.F.P. from the pseudo-absolute scale is less than 0·03°C. Even at the boiling point of water under standard conditions the discrepancy between these two scales is only 0·05°C., and, therefore, only a third of the uniform dis-

crepancy between the absolute centigrade and the pseudo-absolute scales.

The I.F.P. scale has for its fundamental fixed points zero and 273. The boiling point under any specified pressure may be regarded as a secondary fixed point and used for calibrating thermometers. As will be seen from the table, the temperature 373 may be fixed as the boiling point under 1,015·1 mb.

The general adoption of the absolute scale for popular use in the near future is not to be anticipated, but there seems to be no good reason why it should not find a place in the physical laboratory at once. Thermometers are always being broken and replaced, and therefore the substitution of the absolute for the centigrade scale may well be a gradual process.

ABSTRACT.

The Paper urges the general use of the new scales of pressure and temperature which have been adopted by meteorologists. In the pressure scale the fundamental unit is the bar, the pressure due to a million dynes per square centimetre. The practical unit is the millibar. The temperature scale is that known as the pseudo-absolute scale, obtained by adding 273 to the centigrade scale. The author, however, considers that it would be advantageous to use the "Integral Freezing Point" scale, in which the interval between absolute zero and the freezing point of water is divided into 273 degrees exactly.

DISCUSSION.

Dr. C. Chree thought the millibar was a convenient unit for practical use He did not think too much stress should be put on the existing estimates of the absolute zero, which might be in error by more than was expected. He remembered how at one time the temperature 62°F. was fixed in reference to a particular Kew thermometer made of an unusual kind of glass, and used horizontally although it had been calibrated vertically.

Mr. F. E. Smith said that in reference to the remark about National Physical Laboratory certificates, these certificates always referred to the hydrogen scale, and there was in this case no ambiguity as between absolute or psuedo absolute.

Mr. Whipple said he had not noticed that any of the certificates he had seen specified the zero.

Prof. Lees asked if the author was definitely recommending the use of the I.F.P. scale rather than the other.

Mr. Whipple: Yes.

Prof. Lees then said that it was easy to get the correction from the pseudo absolute to the absolute centigrade scale by simple addition; but if the I.F.P. scale were adopted the conversion would be somewhat difficult, especially if the accepted value of absolute zero had ever to be revised.

XVIII. *On the Transmission of Speech by Light.* *By* A. O. Rankine, *D.Sc., Fellow of and Assistant in the Department of Physics in University College, London.*

Communicated by Prof. W. H. Bragg, F.R.S.

Received April 15, 1919.

Note.—The experiments described in this Paper were carried out between February and October, 1916, at the request of the Admiralty Board of Invention and Research. The results are now published by permission of the Admiralty.

Introduction.

The notable property of selenium of varying its electrical conductivity when exposed to illumination of various intensities has long been well known. It has led to various attempts being made during the last thirty or forty years to transmit speech over considerable distances by means of a beam of light which fluctuates in intensity in a suitable manner. Given such a beam of light the mode of reproduction is simple. A circuit is made consisting of a selenium cell exposed to the beam, a telephone receiver, and an electric battery. If the intensity of the beam does not vary, the electric current through the telephone receiver remains constant. But, if there have been impressed on the beam fluctuations of intensity corresponding in amplitude and frequency to the vibrations of speech or other sounds, the selenium, if it is capable of adjusting its conductivity with sufficient rapidity, will control the current in the telephone in such a way as to reproduce the original sounds.

This Paper is concerned chiefly with the manner in which it is possible to produce a beam of light, fluctuating in accordance with speech sounds. The methods hitherto used may be divided into two classes. In the first, the aim is to cause the speech to control the illuminating power of the source itself. For example, if the current in an electric arc can be controlled effectively by microphonic action, the light issuing from the arc may be expected to have the character desired. The second general method is to effect the control of the beam by causing the speech to interrupt the light, with the proper periodicity and amplitude, *after* it has left the source, the actual illuminating power of which remains constant.

Of these two modes, there is little doubt that the second is the more effective and useful. In the first place, it permits the use of the sun's light as the source, whereas, in the other method, artificial sources only can be used. This, it will be seen later, is of considerable importance. It is desirable, also, that the changes of intensity brought about by the speech should be made as great as possible. In the case of an arc controlled microphonically, however, even if the current oscillates from zero to its maximum value—which is unlikely—the variations of light intensity must still be comparatively small at the frequencies in question. If we take the average frequency of speech sounds as about 500 per second, it means that the brightness of the arc must alternate between maximum and minimum every 1/500 second, during which time the actual variation of its temperature, and, therefore, of its brightness, must be small. Speech would thus impose no more than a ripple of small amplitude upon the already powerful beam of light emitted from the arc. On the other hand, by controlling the beam after it has left a constant source, it is possible, particularly by the method about to be described, to guarantee that the fluctuation of the beam traverses the widest possible range.

Both methods have been used with a certain amount of success. In a patent specification of 1889, Graham Bell describes two devices falling under the latter class. In the first, the proposal is to allow the beam of light to pass in succession through two grids consisting of equal parallel strips alternately opaque and transparent. One of these is fixed in position, and the other is moved bodily, in a direction perpendicular to the strips, by the operation of a diaphragm to which it is rigidly attached, and on which speech sounds fall. The movements of the diaphragm may be expected, therefore, to control the obstruction to the beam of light in such a way that the intensity of the emergent beam varies in accordance with the speech sounds. The practical objection to this device is that, ordinarily, the movements of the diaphragm are so small that it would be most difficult, if not impossible, to make grids at once so light, so rigid and so fine as to fulfil the necessary conditions. Graham Bell does not, in fact, claim that this method has been actually used. His second device is much more sound from a practical point of view. It relies upon the fact that a vibrating diaphragm is continually altering its curvature; consequently, if polished, and

interposed in a steady beam of light, it will give rise to a reflected beam which is of variable divergence. Although the total amount of light reflected is actually unaltered, the fraction of it incident at a distance upon a receiver, not large enough to include the whole beam, will have a value controlled by the vibrations of the diaphragm. The objection in this case is that the diaphragm must be large in order to deal with a large quantity of light; for, if it is placed at a point where the beam has been concentrated to a focus, the effect of its changes of curvature upon the divergence of the beam is negligible. Further, from an acoustical point of view, it would be necessary for the diaphragm to be thin, and it would, therefore, not be suitable for adaptation as a mirror of good optical properties.

In January, 1916, Prof. W. H. Bragg asked the author to investigate the problem of producing a device for controlling by human speech the intensity of a beam of light, and at the same time suggested the former of Graham Bell's methods, being, however, unaware that the device had been proposed previously. The method about to be described was the outcome of this suggestion. It has proved to be a practical and reliable means of transmitting speech by light, and ranges up to $1\frac{1}{2}$ miles have been achieved, in spite of the fact that, owing to war conditions, it has not yet been possible to construct the apparatus in its most convenient and efficient form. There appears, also, to be room for considerable improvement in the selenium cells which have been used as receivers, and it is quite probable that the range can be increased to several miles.

Description of the Apparatus.

In this apparatus the essential point is the substitution of the *image* of a grid for the material grid itself. This substitution at once surmounts all the difficulties inherent in Graham Bell's first proposal. It is no longer necessary to attempt to construct diminutive grids suitable for being mounted upon, and operated by, speech-receiving diaphragms. *Both* grids are now fixed in position, and may be of practically any size. It is the *image* of the first grid which is caused by the incident speech to move to and fro with respect to the second grid. The principle involved will be readily seen by considering the first diagram (Fig. 1). *A* is a small mirror—an ordinary galvanometer mirror of about 1 cm. diameter—

which can be caused, by means to be described later, to execute small angular oscillations about an axis perpendicular to the plane of the diagram. Light from a source S is concentrated upon it by means of the lens L—*i.e.*, an image of the source is formed upon it. The equilibrium position of A is arranged to be such that the divergent reflected beam proceeds to a second lens L_2, by which it is brought to a focus at F. A grid G_1, consisting of parallel strips alternately transparent and opaque and perpendicular to the plane of the diagram, is interposed in the position shown, close to the lens L_1; and a second grid G_2 of dimensions equal to the first is placed in the path of the beam reflected from A in a position such that $AG_2 = AG_1$. The small mirror A is a concave one of radius AG_1. Consequently, leaving out of account for the moment the effect of obliquity, a real image of the grid G_1 is formed

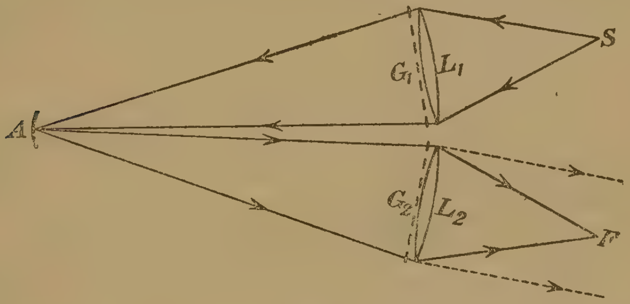


Fig. 1.

in the plane of the grid G_2. It will be noticed that, by reason of the fact that an image of the source of light is formed upon the mirror A, the curvature of the latter exercises no control on the divergence of the beam as a whole; what it does is to secure that *all* the light passing through any point in the plane of the first grid passes also through a corresponding point in the plane of the second one. It is clear that for a certain angular position of A the image of the first grid will be so situated on the second grid that, being of equal dimensions, the images of the opaque portions will become exactly superimposed on the transparent portions of the second grid. In this case no light at all will emerge from the system, to be focussed at F. For another position of A, the non-luminous portions of the grid image fall exactly on the opaque portions of the second grid, and the luminous portions on the trans-

parent slots; so that, apart from absorption, 50 per cent. of the original beam of light is brought to a focus at F. In general, A will be so placed that a fraction of the light varying between 0 and 50 per cent. of the original amount will penetrate the complete system. It will now be clear that if A has imposed upon it angular oscillations of any frequency and amplitude whatever, provided only that the amplitude is never greater than that corresponding to the width of one slot of the grids, the emergent beam will have a total intensity which is equal in frequency to, and proportional to the amplitude of, the oscillations of A. The image of the first grid, in moving to and fro over the second grid, in effect opens and closes a shutter in the path of the beam.

It is important to notice that for efficient interruption it is essential that the small mirror A should be a concave one, producing on G_2 a sharply defined image of G_1. Any source of light, if it emits luminous energy at all, has, of necessity, finite size. If A were plane, only a single point of the source would be provided for; to provide similarly for the *whole* of the light, the two grids must be at conjugate points with respect to A. It is not necessary, of course, that the grids should be equi-distant from A; but if their distances are not equal, they must still be such that the image of G_1 is focussed on G_2, and, in addition, the dimensions of G_2 must be adjusted so as to be identical with those of the image of G_1.

There are several ways in which the small mirror A may have imparted to it oscillations corresponding to human speech. Experience so far shows that, perhaps, the most direct one is the best. The mirror is attached rigidly to the lever which ordinarily carries the needle in a good quality sound box or gramophone recorder. The manner of attachment is indicated in the diagram (Fig. 2). Speech sounds traversing the conical trumpet impinge on the mica diaphragm, causing the lever to vibrate and to impart its angular motion to the mirror. Only small angular oscillations are possible by this means, the order being about 1/400 radian. But it is easy to see that by making the distances AG_1 and AG_2 (Fig. 1) large, and the spacings of the grids themselves of comparatively small dimensions, the speech sounds can be made to control efficiently the movements of the grid image, and, hence, the quantity of light emerging from the system.

A selenium cell placed at F so as to receive the emergent light, and, connected in series with a suitable telephone receiver and

battery, serves as the means of reproducing sounds corresponding to the fluctuations of the light. For speech sounds, with sound box and selenium cell of good quality, the articulation of the reproduced speech is extraordinarily good. It is interesting to note in passing, however, a striking effect which can be anticipated and which does actually occur in practice. Suppose that the small mirror A has imparted to it simple harmonic oscillations of a definite frequency n, but of variable amplitude. A tuning fork as the source of sound might, for example, be used for the purpose. If the amplitude is so small that the extreme movements of the grid image do not extend to more than the width of one space of the grid G_2, the intensity of the emergent light has the same frequency n, and this frequency is reproduced in the telephone receiver. An increased amplitude will, however, result in the light suffering interruption *twice* for each complete oscillation of the mirror, and the octave $2n$, in addition to the original note, will be produced. With still greater amplitudes the successive harmonics $3n$, $4n$, &c., will be added, and the original note may thus give rise to a whole series of overtones. When transmitting speech this effect makes itself evident by transforming an ordinary word into a screech. It can be avoided by modulating the voice so that the grid image, which is visible to the speaker, does not move over an excessive range. This is found to apply much more to some words than to others. "Four" and "five," for example, are very effective in their action on the diaphragm, and must be spoken comparatively softly; "two" and "three" produce feeble effects, and may be spoken more loudly.

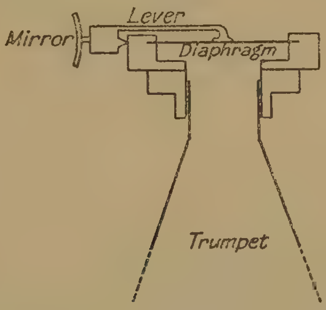


FIG. 2.

When it is desired to project the fluctuating beam effectively to a distance, it is necessary that the vibrating mirror A should be at the focus of the optical system—in this case of the lens L_2. The light from every point of the source is then made into a parallel beam, as indicated by the dotted lines in Fig. 1, and the whole beam spreads at an angle determined by the ratio of the diameter of the small mirror A (supposing this mirror is completely covered with light from the source) to the length AL_2. The amount of light received on a limited area diminishes as the area is removed further from the projector. A lens or mirror of as large an aperture as convenient is placed so as to receive part of the projected beam, and the selenium cell is placed near the focus. The arrangement for this purpose is shown in Fig. 3, where also

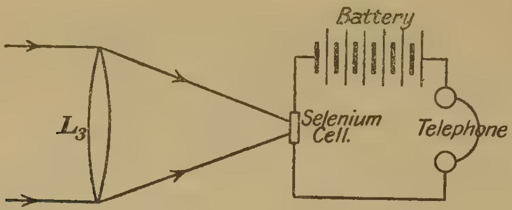


Fig. 3.

are represented the electrical arrangements for listening. It turns out to be best not to put the selenium cell precisely at the focus of the collecting lens L_3, but to place it so that the converging beam just covers the whole of the sensitive area of the selenium.

There are many alternatives in the optical arrangements of both the transmitting and receiving systems, and in sources of light. The question of the relative advantages of such factors will be dealt with later. One particular arrangement of the transmitting system (probably not the best) is shown in the photograph (Fig. 4). This particular apparatus has been made up from stock sizes of the various parts, owing to the difficulty of getting optical apparatus made to specification. It is shown set up for use with the sun as source of light. The large sun reflector—on the left—is a plane mirror of 9 in. diameter. It projects the light received from the sun, first through a water cell to remove most of the thermal radiation, then through the first lens and grid, so that an image of the sun is focussed on the small oscillating mirror,

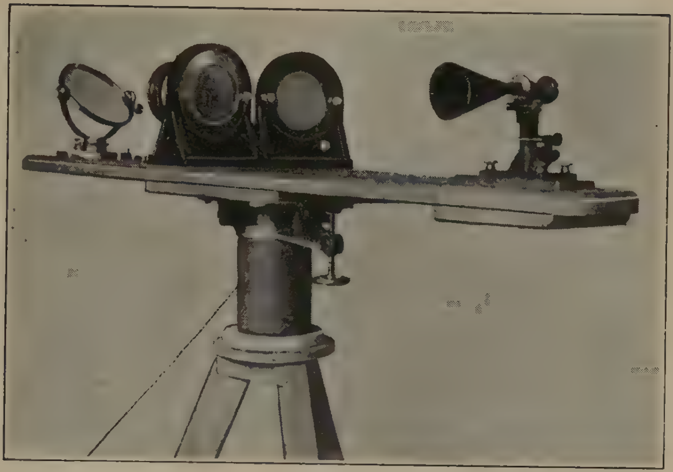


Fig. 4.

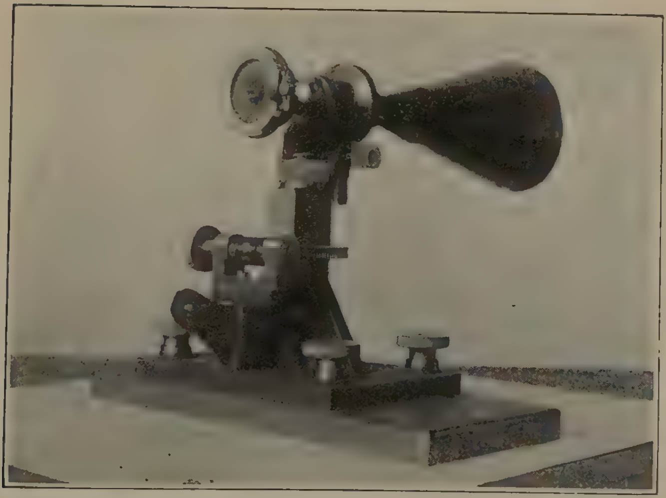


Fig. 5.

[To face p. 248.

which is actuated by sounds spoken through the conical trumpet on the right. The light diverges and passes through the second grid and lens, and is projected as a parallel beam to a distance. Each of the lenses has a focal length of 1 yard, and the radius of curvature of the small oscillating mirror is also 1 yard. The lenses and the grids have apertures of about $5\frac{1}{2}$ in., and the alternate opaque and transparent bands are each 0·1 in. wide. The apparatus has an altazimuth mounting and is provided with sights for directing it on to the receiving station. The sun reflector must be kept in continuous adjustment so that the image of the sun remains on the oscillating mirror. Artificial light can also be used. In this case the source takes the place of the sun reflector, and a lens of appropriate focal length has to be substituted for that used with the sun in order to produce an image of the source upon the oscillating mirror.

The mounting of the sound box and oscillating mirror is shown in greater detail in the photograph (Fig. 5). Provision is made for screw adjustment of the oscillating mirror about vertical and horizontal axes for setting the image of the first grid in correct position on the second. The trumpet is made of thick drawing paper, this having been found much superior to tin, which imparts metallic characteristics to the speech sounds.

With the above apparatus as transmitter and with selenium cells of the type available as receivers, speech is audible at considerable distances. Although the collecting lens used was only 7 in. in diameter, a "pointolite" lamp suffices up to a distance of half a mile. With a carbon arc as source, the range is considerably greater; whilst the sun multiplies the distance many times, probably to several miles.

Alternative Arrangements of Apparatus.

It is not proposed to enter fully into the description of the considerable variety of possible alternative arrangements embodying the principle of the apparatus already described. Several of these have been tried successfully. In the transmitting system, for example, it may be convenient to substitute for the lens L_2 (Fig. 1) a concave mirror of equal focal length. Other variations will be sufficiently obvious. An auto-collimating arrangement deserves especial mention, for most of its features seem to be ideal, at any rate in the case where the source of light is artificial. It enables one of the

grids to be dispensed with, and the beam of light is interrupted by means of the movements of the image of a single grid upon that grid itself.

A plano-convex lens, silvered internally upon the flat face behaves as a concave mirror, the radius of curvature of which is equal to the focal length of the lens. If, instead of silvering the whole of the flat face, we silver it only in parallel strips separated by equally wide unsilvered spaces, it will combine the properties of a lens with those of a mirror. Light incident upon its curved surface will be divided into two practically equal portions, one part being transmitted, and the other part reflected. Suppose we have such a lens (illustrated in section PQ in Fig. 6), and a source of light S removed a little from the axis OX of the lens, but in the focal plane. Half of the incident beam PSQ will be transmitted through the transparent spaces of the lens, and projected as a parallel beam represented by the lines PP_1, QQ_1, making with the

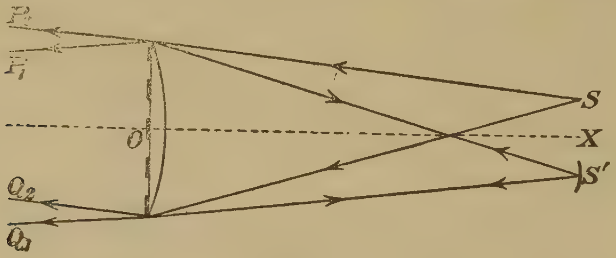


FIG. 6.

axis of the lens an angle equal to SOX. With this beam we have no further concern. The remainder of the beam PSQ, on the other hand, will be reflected by the silvered strips, and will converge to a focus at S^1. An image of S will thus be formed symmetrically placed with S itself relatively to the axis of the lens, both S and S^1 being in the focal plane. Now, if a small concave mirror whose radius of curvature is equal to S^1O be placed at S^1, and be actuated by a sound box in the manner already described, it can be used to produce a real image of the silvered grid upon this grid itself. The movements of the small mirror at S^1 thus determine the quantity of light which penetrates this grid, and a parallel beam of fluctuating character (represented by the lines PP_2, QQ_2) is projected in a direction making an angle equal to S^1OX with the axis of the lens, and twice

this angle with the non-fluctuating beam PP_1, QQ_1. It is easy to arrange that these two beams separate sufficiently rapidly to be entirely distinct at considerable distances; and it is merely a matter of adjusting the sighting arrangement accurately to guarantee that it is the fluctuating beam which reaches the point desired.

This arrangement works quite satisfactorily. It was, however, a difficult matter to make an accurate grid by hand, the method being to cut and remove strips from the lens surface which had been originally silvered all over. Suggestions for the more efficient construction of this type of lens are given later, and there appears to be little doubt that this arrangement will ultimately be superior to the other alternatives.

Alternative to Gramophone Sound Box.

From some points of view it may be desirable that the speaker whose words are to be transmitted should not necessarily use the trumpet mounted on the apparatus as shown.

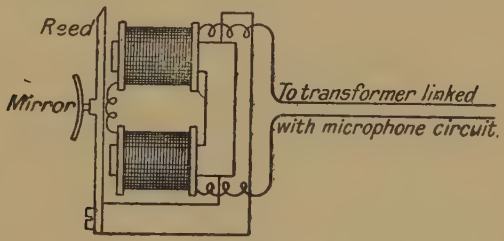


Fig. 7.

This may be provided for by substituting for the gramophone sound box a telephone receiver which is actuated by the variable current in a microphone to which the words are spoken. For this purpose a telephone receiver earpiece of the reed type is preferable, for the reed executes angular oscillations in response to alternating currents in the electromagnet. The small concave mirror (Fig. 2) is now attached rigidly to the reed of the telephone receiver, as shown in Fig. 7. The coils of the receiver are connected to the secondary of a suitable transformer, the primary of which is in series with a battery and the microphone. Sounds impinging on the diaphragm of the latter thus result in movements of the mirror which control the intensity and frequency of the light projected. This

arrangement has been tried and proved successful. In this case the microphone and speaker may be quite remote from the light transmitter. It has been found, however, that the intervention of the additional electromagnetic transformation of the speech sounds, results in a considerable loss of perfection in articulation.

Conditions of Efficiency.

It is now proposed to discuss in detail the various factors which determine the efficiency of the transmission of speech by the means described. Selenium is, apparently, sensitive to all the constituent parts of light forming the visible spectrum. Hence, it is clearly desirable that as great a quantity of light of all possible wave-lengths should be transmitted to and collected at the receiving end, and that all this light should be effectively controlled in intensity and frequency by the speech sounds. The selenium cell, too, should be constructed so as to be as sensitive as possible to rapid fluctuations of light.

(*a*) *Source of Light.*—If we make the assumption that the small oscillating mirror is covered completely with light by the image of the source formed upon it, the amount of light transmitted (apart from the small amount of absorption by the various mirrors and lenses) is determined solely by the brilliancy of this image. If the light forming this image is collected from the source by a lens or mirror of fixed aperture, the brilliancy of the image and the light projected to a distance are proportional to the intrinsic brilliancy of the source. Experiments have been made with three different types of source—a "pointolite" lamp, a carbon arc, and the sun. A rough comparison has been made of the relative intrinsic brilliancies of the "pointolite" and the sun. The method used was to form an image of each close together, by means of two separate lenses of equal focal length, and to diminish the amount of light received from the sun by applying various diaphragms until the intensities of the images were judged to be equal. The intrinsic brilliancies of the two sources were then in the inverse proportion of the areas of the respective apertures. Some difficulty was experienced in the comparison, owing to the difference of colour of the two images, but it may be taken that the ratio 250 : 1 fairly represents the relative intrinsic brilliancies of the sun and the "pointolite." The carbon arc is intermediate between the two, but much nearer the latter. It has not yet been possible actually to

compare the ranges for transmission of speech attainable with these various sources. Neglecting absorption it may be expected that the range for the sun would be $\sqrt{250}$—*i.e.*, 16 times as great as that for the "pointolite." The carbon arc would probably give two or three times the range of the latter. Although the "pointolite" is the least efficient it has one great practical advantage. It is a source of unvarying position. The carbon arc requires continual adjustment, and it is difficult to keep the image of the hottest parts of the arc (*i.e.*, the positive crater) steadily on the oscillating mirror. The same objection applies, although not to the same extent, to the use of the sun as source. An efficient automatic heliostat would overcome this difficulty; and, in addition to the great range attainable, the use of the sun as source does not involve any electrical equipment at all being transported with the transmitting apparatus.

(*b*) *Optical System.*—In the transmitting apparatus the two important constituents of the optical system are the oscillating mirror and the projecting lens or mirror. If conditions are arranged so that both are filled with light, the total quantity of light transmitted is determined by their apertures, and is in proportion to their areas. In the receiving system, the size of the receiving lens or mirror controls the amount of light collected and concentrated on the selenium cell. But it is not practicable to have it so large that it will collect at great distances more than a comparatively small fraction of the light projected by the transmitter. Consequently the diameter of the oscillating mirror, which merely controls the rate at which the projected beam diverges—and not its intensity—is comparatively unimportant, except in relation to the special consequences of this divergence. For example, in the actual apparatus already described, the ratio of the diameter of the oscillating mirror to the focal length of the projecting lens is about 1/100, so that the divergence of the projected beam is 0·01 radian. The diameter of the illuminated disc at a distance of a mile is 18 yds, and only a small fraction is collected by a receiver of reasable dimensions. The diameter of the oscillating mirror might, therefore, be much smaller without reduction of efficiency; but such diminution would require greater accuracy in directing the transmitter.

The oscillating mirror should be as perfect as possible so as to produce a clearly defined image of the first grid. The ideal

S 2

condition is that there should be no diffused light, so that for the appropriate position of the grid image on the second grid, no light at all should be transmitted. Care has to be taken to mount this mirror so that it is free from strain. If it is silvered on the back surface, it is usually found that the faint image from the unsilvered front surface has a position somewhat different from that of the main image. This can be remedied by rotating the mirror about its principal axis until the two images are in alignment, although not actually superimposed.

The intensity of illumination at a distant point is directly proportional to the area of the projecting lens or mirror, supposing the latter to be optically perfect. The larger this projector is, the greater will be the range. Unwieldiness is the ultimate limitation. For, in addition to having a large aperture, the projector should, for two reasons, have a correspondingly great focal length. In the first place, the angular amplitude of the oscillating mirror produced by speech of ordinary intensity is very small (of the order of 1/10 deg.). For efficient interruption of the light the width of the grid spacings must be small, and the distance between the oscillating mirror and the second grid large. In practice it has been found difficult to make grids of sufficient accuracy less than 0·1 in. wide, and, in such circumstances, for speech without inordinate effort, the focal length of the projecting lens should not be less than 1 yd. In the second place, spherical aberration troubles can only be avoided by keeping fairly large the ratio of the focal length to the diameter of the projector. It may be that the ratio usually adopted in astronomical instruments (about 12 : 1) is unnecessarily large for the present purpose, although it may be mentioned that very good results have been obtained using an 8 in. reflector of 7 ft. focal length, kindly lent by the Royal Astronomical Society. But a ratio of not less than 6 : 1 is almost certainly necessary for efficient transmission. With a projector 12 in. in diameter, the length of the apparatus would be in excess of 6 ft.—a size already somewhat unwieldy, apart from special provision as regards mounting.

The question as to whether a lens or a mirror should constitute the projector is a debatable one. Both have been used. A mirror has the advantage that it is free from chromatic aberration, but its efficiency is reduced by the fact that either it must be mounted obliquely so that the projected beam may

not encounter the sound box and trumpet, or one must allow the amount of light projected to be reduced considerably by the intervention of these objects. An ordinary lens loses light by chromatic dispersion, although it has the advantage that there is no obstruction when mounted normally to the oscillating mirror. The experience of the author has led him to prefer the lens, particularly if it is corrected for chromatic effects, because it leads to the single grid system already referred to.

It is interesting to make a calculation showing what gain may be expected from the use of achromatic lenses. Let us suppose that the projecting lens is uncorrected and that the oscillating mirror is placed at its focus for the extreme red component of the spectrum. Let F be its focal length for this colour, D its diameter, and d the diameter of the oscillating mirror. Then at a distance x from the transmitting lens, the diameter of the projected beam (Δ) is given by

$$\Delta = D + x \cdot \frac{d}{F},$$

$\frac{d}{F}$ being the divergence of the beam, if the whole of the oscillating mirror is covered with light. At great distances D becomes negligible, so that

$$\Delta = x \cdot \frac{d}{F} \text{ approximately.}$$

This gives the inevitable normal divergence attributable to the size of the source. If the lens were perfectly achromatic the other components of the light would also be confined within this disc. As it is, however, each of these components has an additional divergence, owing to the variation with colour of the focal power of the lens. If the focal length for the violet extreme of the spectrum differs by δF from F, the additional divergence φ in this case is

$$\varphi = \frac{\delta F}{F} \cdot \frac{D}{F}.$$

$\frac{\delta F}{F}$ is the dispersive power of the material of the lens over the range of colour specified. Calling this P we have that Δ_1, the

total diameter of the disc over which the violet light is spread at a great distance x is represented by

$$\Delta_1=\frac{xd}{F}+\frac{xD}{F}\,.\,P \text{ approximately.}$$

Whence $$\frac{\Delta_1}{\Delta}=1+\frac{D}{d}\,.\,P.$$

The ratio of the intensities of the violet and red components respectively is equal to

$$\frac{\Delta^2}{\Delta^2_1}=\frac{1}{\left(1+\frac{D}{d}\,.\,P\right)^2}.$$

It will be seen that, especially if the ratio of the diameter of the projecting lens to that of the oscillating mirror is large, the violet light may be spread over a much greater area than the red, and, in consequence, be less effective on the selenium. In the apparatus previously described the projecting lens is of crown glass for which $P=0{\cdot}043$, and the ratio $\frac{D}{d}=17$.

Thus $$\frac{\Delta^2}{\Delta^2_1}=\left(\frac{1}{1{\cdot}73}\right)^2=\tfrac{1}{3} \text{ approximately.}$$

For colours intermediate between the red and the violet, the reduction of efficiency above indicated will be less marked, but still considerable, and it is clear that the use of achromatic lenses would result in a notable increase of light on the selenium receiver.

In the case of the single-grid system, it is necessary that the strips should be reflectors. In the apparatus of this type which has been tried, the grid was formed by cutting and removing strips of silvering from the plane surface of the lens which had been previously silvered all over in the ordinary way. It deteriorated rather rapidly owing to the corrosion of the edges of the strips.

This might be avoided by depositing the film by kathodic bombardment through a template in the form of a grid. It would probably be advantageous to protect the grid by a sheet of plane glass attached by Canada Balsam. The author believes that an achromatic lens, of optical quality approaching that of a telescope objective, treated in the way above described, would form the most efficient form of the apparatus. Such a lens, say, of 8 in. aperture and 4 ft. focal length, com-

bined with a oscillating mirror of 4 ft. radius of curvature, would constitute a transmitter of convenient size.

The remaining part of the optical system is that at the receiving end. Here, either a lens or a mirror may be used, and either should have a large aperture in order to concentrate as large as possible a fraction of the transmitted beam on the selenium. Optical quality is not so important as in the transmitter, but it still deserves attention. It is significant that, in practice, it has been found that an ordinary 7 in. lens forms as efficient a receiver as a 24 in. searchlight mirror. Although in the former case the selenium cell was shielded from all light save the transmitted beam, and consequently, the cell itself operated more efficiently, the discrepancy cannot be wholly accounted for in this way. A large part of it is undoubtedly due to the inaccuracy of the reflecting surface of the searchlight mirror, resulting in the waste of a large fraction of the incident light.

(*c*) *Grids.*—Very careful attention has to be given to the construction of this part of the apparatus. For reasons already indicated it is necessary that the spacings should be quite narrow. Two things are essential. In the first place the image of the first grid must be of the same dimensions as the second grid. It is clear that if they are out of step by one spacing, the fluctuation of intensity of the whole emergent beam will be reduced to zero, for the beam will then consist of two equal halves fluctuating in opposite phases. A difference of even a small fraction of the width of a spacing is detrimental, although not to the same extent. The second essential is that the grid image and the second grid must be strictly parallel to one another. If the two intersect only once there is no resultant variation of the total light transmitted. It has been already pointed out that, theoretically, the two grids may occupy any pair of positions conjugate with respect to the oscillating concave mirror, provided their sizes are appropriate. But as a consequence of the above conditions, it has been found that the most satisfactory results are obtained with the distances from the oscillating mirror equal; for in this case, the two grids have to be equal in size. A pair may, therefore, be milled together out of superposed sheets of metal, so that, whatever may be the inaccuracies of milling, the two are of practically indentical shape. The second is used inverted with respect to the first, and coincides with its inverted image. This is only so with sufficient accu-

racy when the angle is small through which the whole beam has to be turned by the oscillating mirror in order to penetrate the projecting lens. Thus, in Fig. 1, if the angle G_1AG_2 is considerable, the light falls *obliquely* on A, and the image formed by this mirror is both reduced in size and nearer to A than it would have been for normal incidence. This is an additional reason for keeping the ratio of the diameter of the lenses to their focal lengths comparatively small. Should the effect of obliquity become serious, it may be largely reduced by arranging that the grids are parallel, instead of being perpendicular, to the plane G_1AG_2, and by providing that the oscillating mirror is caused to vibrate about the appropriate axis—*i.e.*, a horizontal one. The grids are then parallel to the secondary and not the primary focal line, and the position of the former is not so sensitive to increase of obliquity as in the latter case. This effect of obliquity disappears almost entirely in the case of the single grid system, since the light is parallel to the axis of the oscillating mirror.

However accurately the grids are made in practice their edges are bound to suffer from some degree of imperfection. The normal position of the oscillating mirror is arranged so that the images of the edges of the first grid coincide with the *middles* of the strips of the second grid. The intensity of the speech should then be modulated, so that the amplitude of vibration is never so great that coincidence of edges quite takes place. The transmitted beam, averaging 50 per cent. of that leaving the oscillating mirror, will then have an intensity which fluctuates in strict accordance with the vibrations of the diaphragm, even if the edges are inaccurate, provided the edges of image and second grid never cross. The *mean* intensity of the projected beam is the same whether it is fluctuating or not. The eye is, of course, unable to perceive the separate fluctuations of speech frequency, so that an observer would see an apparently steady light, and that only if he happened to be within the comparatively small angle of divergence of the beam.

To start with the grid image and second grid in precise register gives rise to three disadvantages. These have been encountered in practice, and it is easy to see the reasons for them. If one arranges that the normal amount of light transmitted is as near zero as possible—*i.e.*, if the images of the slots of the first grid are normally exactly superimposed on the opaque strips of the second—the accuracy of the edges

becomes important. Any departure from perfection results in the variation of intensity of the beam not being in strict proportion to the change of displacement of the oscillating diaphragm. This shows itself in a loss of articulation in the words transmitted. Further—and this is still more important in its effect on faithfulness of reproduction—each single vibration of the diaphragm produces *two* (instead of one) maxima of intensity in the projected beam, resulting in the formation of octave overtones in the sounds received. With this setting, also, every word spoken into the trumpet causes the mean intensity of the projected beam to be no longer zero ; successive flashes, corresponding to each syllable, are seen, and it is evident to an outside observer that signalling of some sort is proceeding. From all points of view, therefore, the "half" setting first described is definitely preferable.

The effect of excessive amplitude of vibration in relation to width of grid spacings has been already referred to. For speech of a constant intensity, the amplitude of the light fluctuations is increased by reducing the width of the grid spacings. Or, looked at from the other point of view, feeble speech sounds combined with sufficiently narrow spacings, can produce effects equal to those arising from louder speech with wider spacings. The spacings should be as narrow as is consistent with accurate construction. In such circumstances, and with a bright source of light, the transmitter may actually act as a relay, the sounds heard in the telephone receiver being more easily audible than those incident on the transmitter. This effect has been actually observed at considerable distances, when the sun has been used as source.

(*d*) *Sound Box.*—The sound boxes used have been those of the "Exhibition" grade, made by the Gramophone Company, Ltd. They appear to copy sounds most faithfully and have given excellent results. It has been suggested that for the present purpose a less stiff diaphragm might be used so as to make possible the use of grids with wider spacings, or, alternatively, to produce equal effects with feebler sounds It is very doubtful, however, whether the gain of this advantage would compensate for the possible loss of two others. Stress has already been laid on the importance of maintaining the "half" setting of the grid image, and this is best secured by a stiff mounting for the oscillating mirror. In addition, a stiff diaphragm is undoubtedly more nearly aperiodic for speech frequencies than a more flexible one, and is consequently a

more faithful reproducer. Altogether, the above sound boxes are so good that there is little room left for improvement. The articulation of the speech heard at the receiving end is already so clear that there is little or no difficulty in recognising the identity of a speaker whose voice is familiar to the listener. This fact proves not merely the excellence of the transmitting sound box, but also that of the telephone receivers which are in series with the selenium. These have been of the reed type manufactured by S. G. Brown, Ltd. The possibility of the substitution of one of these receivers (in conjunction with a microphone) for the sound box has already been mentioned. It is practically certain that the loss of articulation in this case is attributable to the microphone, and not to the telephone receiver.

(*e*) *General Mounting of the Apparatus.*—For convenience of transmitting it is clear that all the parts of the transmitter should be mounted on a rigid base which is capable of being rotated about horizontal and vertical axes. The mounting required is, in fact, similar to that necessary for a gun. For most purposes ordinary pin-hole sights give sufficient accuracy in directing the beam, but telescopic sights might be necessary for great distances. It is best to adjust the sights or telescope in practice by causing the projecting lens to form an image of a distant object on the oscillating mirror, upon which it can be readily seen even in daylight, and then setting the sights or telescope in line with the object. There is, of course, a good deal of latitude permissible in the sighting, depending on the divergence of the projected beam. An accuracy of only half a degree, for example, is necessary in the transmitter shown in Fig. 4. As it is *angular* accuracy only which counts, there is no increase in difficulty of pointing as the distance increases.

The receiving apparatus also should be similarly mounted. In a double set the act of sighting the transmitter would automatically put the receiver in adjustment for receiving.

(*f*) *Selenium Cells.*—The selenium cells used in these experiments have all been of the type due to and made by Dr. Fournier D'Albe. Those giving the best results have had a resistance (when not illuminated) of from 10^5 to 2×10^5 ohms. A pair of Brown receivers, wound to a total resistance of 6,000 or 8,000 ohms proved the most satisfactory. A transformer may be used so as to avoid any continuous

component of the current in the receivers, but this is somewhat detrimental to articulation—probably owing to the hysteresis of the iron core of the transformer. A satisfactory substitute for a transformer is the arrangement shown in Fig. 8. In series with the selenium cell is put a non-inductive resistance R, the resistance of which equals that of the selenium cell under the conditions of use. The ends of this resistance are tapped (through a condenser) by the telephone receivers. In this case variations of current only pass through the latter, and, by choosing the condenser so that its impedance for speech frequencies is very small compared with that of the telephone receivers, the articulation can be rendered not noticeably inferior to that in the direct arrangement.

There is no doubt that these selenium cells are capable of responding sufficiently faithfully to the rapid fluctuations of the transmitted beam to give surprisingly good articulation. They appear to retain this property over long periods, particularly if not in regular use. Some of them, manufactured three years ago, are still good, although not probably quite so efficient as they were originally. Constant exposure to bright light does, however, produce slow deterioration, especially in combination with a high voltage. The limit permissible is apparently about 40 volts; otherwise the cell rapidly breaks down and ultimately becomes practically non-conducting. It is significant that the above voltage is the arcing potential of carbon, and it seems quite likely that, above this voltage, arcing between the graphite layers forming the basis of the cell sets in and burns out the selenium.

There is also evidence that the cells work better when shielded from bright extraneous light; and a cell, which it is desired to keep especially sensitive, should be reserved for use at great distances, when the amount of light falling on it will always be small.

Although these selenium cells have given admirable results, it is certain that there is room for great improvement in their sensitivity to *rapidly fluctuating* light. This is very important, since it limits the range obtainable with the present method of transmitting speech by light. At telephonic frequencies the cells do not respond as quickly as might be to the changes of intensity of the incident light. Suppose that a selenium cell is in circuit with a battery, and that it is exposed to light which can be interrupted at any desired rate. There will be an

alternating current superimposed upon the steady current already in the circuit. So long as the speed of interruption is low, say, 1 or 2 per second, the amplitude of the alternating current will be comparatively large. But this amplitude diminishes very rapidly as the frequency of interruption is increased. The result is that the alternating current obtained at speech frequencies is not nearly so large as it would have been in the complete absence of lag. A good selenium cell whose resistance in the dark is, say, 110,000 ohms, will have its effective resistance reduced by 20,000 ohms (*i.e.*, to 90,000 ohms) by continued exposure to a source of 50 c.p. 1 metre away. Let this cell be in series with a battery of 30 volts and a pair of telephone receivers of 6,000 ohms. The steady current, if the cell is kept dark, will be 30/116,000 amperes=259 microamperes, while that during the exposure of the cell to the light will be 30/96,000 amperes=312 microamperes. Thus, if alternations between light and darkness occur very slowly indeed, the alternating current executes an amplitude of 312−259=50 microamperes approximately. In the complete absence of lag in the selenium, even at speech frequencies (say, an average of 800 per second) the amplitude of the alternating current would still be very considerable, although it would be reduced by the increased impedance of the telephones. These would have at such frequencies an impedance of about 100,000 ohms, so that the current amplitude would be reduced to about 25 microamperes. Such an alternating current as this would produce sounds of almost unbearable intensity in such telephones, which require only a fraction of a microampere to give audible effects. The sounds heard would actually be just about at the lower limit of audibility, proving that the time lag in the selenium has effected a reduction of the current amplitude from 25 microamperes to a small fraction of 1 microampere. The fact that the articulation observed is actually so good is apparently accounted for by the lag being independent of frequency at high values of the latter. If the selenium cells could be so modified as to eliminate or reduce the lag, there would be a considerable augmentation of their efficiency, and of the practical range of transmission of speech. The author has no experience of the methods of manufacture of these cells, but suggests that a diminution of the thickness of the selenium layer might be beneficial.

(*g*) *Possibility of Amplification.*—Even if this improvement in the selenium cells should turn out not to be feasible, there

are still two other methods by which the range can be increased. The first is by increasing the apertures of the transmitting and receiving optical systems. The second is by the use of amplifiers to magnify the alternating currents flowing in the selenium circuit. Of these the second presents less difficulty. Large lenses or mirrors of good optical quality are at present made by processes which are both laborious and costly. Various attempts have been made to use thermionic valves with some success. It soon became evident, however, that this particular type of selenium cell is not capable of being used satisfactorily when the amplification is considerable. Extraneous noises interfere too much. Even when the selenium is kept unilluminated the current flowing in the cell is of an intermittent character which makes itself evident by a loud grating noise in the telephone receivers after triple amplification. This noise is practically inaudible

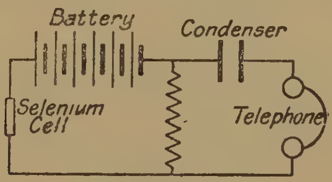


FIG. 8.

in the absence of amplification. It may be that this intermittency of current arises in the selenium itself, but it is more probable that it is caused by the graphite backing which forms the conductor to and from the selenium, since a precisely similar grating noise occurs after amplification of the current passing through a layer of graphite alone, *i.e.*, in the absence of selenium altogether. Whether the current can be rendered more continuous by the substitution of other substances for graphite is a matter for further experiment.

In order to use a valve amplifier with a selenium circuit, it is necessary to provide either a suitable transformer or an arrangement like that shown in Fig. 8, connecting the two telephone leads instead to the filament and grid respectively of the first valve. This is to prevent any voltage, other than the alternating one, operating on the grid and filament. Of the two arrangements the latter is much preferable. In an ordinary three-valve set there are three, or even four, transformers with iron cores. The result is

that in amplifying speech sounds, there is a very great deterioration of articulation, so much so that, although the sounds are very loud the actual words are often quite unintelligible. If, however, non-inductive resistances and condensers are substituted for the transformers, there is a very marked improvement, and this type of valve set has been found to work very satisfactorily with selenium cells, apart from the defect of the latter already referred to.

There appears to be very little doubt that, if selenium cells can be produced which are either more rapid in action, or free from intermittency of conduction except when induced by light fluctuations, the transmission of speech by light will be much facilitated. If both these improvements can be effected, even a comparatively feeble source would probably be capable of transmitting many miles. Already the ranges which have been obtained are quite considerable. With a carbon arc as source, and the projector an astronomical mirror the aperture of which had been reduced to 2 in., speech was quite audible at half a mile. Conversation in both directions has been carried on with 20-in. projectors of poor quality between stations $1\frac{1}{2}$ miles apart, even when considerable mist intervened. With the sun as source, instead of a carbon arc, the faintest whisper could be heard at this distance. All this was without amplification of the sounds received. With satisfactory amplification it is impossible as yet to anticipate how great the limiting range may be.

ABSTRACT.

Light from a point source is collected by a lens of about a metre focus, and an image formed on a small concave mirror, which is attached to the diaphragm of a gramophone recorder. The light diverges and passes through a second similar lens, which projects it to the distant station. Two similar grids are mounted, one in front of each lens. An image of the first grid is superposed on the second by reflection in the concave mirror. When the latter oscillates under the vibrations of speech, the dark spaces of the image move over the openings of the second grid, thus producing fluctuations of the intensity of the beam. The light is received by a collecting lens and focussed on a selenium cell in circuit with a battery and telephone receiver.

DISCUSSION.

Mr. E. H. Rayner thought it a pity that no advantage had been obtained by using a searchlight reflector. At present these mirrors were moulded, and the errors in focus were considerable. Sometimes, however, a really good one could be found by selection.

Prof. Fortescue admired the ingenuity of moving the image of a grid instead of the grid itself, as with any form of microphone the available forces

were small. It appeared that at present very little was known about the selenium cell. It was desirable that physicists should investigate this more fully.

Prof. PORTER commented on the author's thorough investigation of the conditions governing the design.

Mr. C. C. PATERSON asked how much of the sensitivity of the selenium cell was actually effective at these frequencies.

Dr. ECCLES called attention to the historical association of phototelephony with University College, London. Graham Bell's father was a member of the Physics Department at University College some forty years ago, and he himself, there is reason to believe, may have begun there his experiments on phototelephony. The author of the present Paper had worthily upheld the traditional position of the college in this subject. With regard to the present apparatus, the speaker took the opportunity of testifying to its excellent performance in the field. He had witnessed successful trials over a distance exceeding half a mile, and found the articulation much more perfect than is usually achieved with carbon microphones. There are two main types of phototelephony, one in which the source of light is kept constant and the flux of light along the beam modulated by the voice, and one in which the emission of light from the source is modulated. Dr. Rankine's method belongs to the former class. The well-known arc method is illustrative of the latter class, and has been stated to have been used on ships of the German Navy at sea over distances of 7 miles. Its chief disadvantage relative to the first type lies in its poor articulation (so far as it has been developed); its advantage arises mainly through the smallness of the influence of the tremors and shocks always present in a ship, for instance, against which the mobile mirror in Dr. Rankine's apparatus could be protected only with great difficulty. Nevertheless, as an improvement on Graham Bell's grid method, Dr. Rankine's conception of moving a light mirror instead of a heavy grid was an elegant and practical step.

Mr. T. SMITH: I have been greatly interested in Dr. Rankine's Paper, and wish to express my admiration for the way in which he has overcome the difficulties inherent in any problem involving the reproduction of effects at a considerable distance. I propose to limit any comments I have to make to the discussion of the optical problem involved. This is by no means an easy one, and the author is to be congratulated on the efficiency of the arrangement he has adopted. I am not, however, in agreement with much that he has said concerning the optical system in discussing the conditions of efficiency, and I consider that some of his conclusions need modification. Referring to Fig. 1 of the Paper, and disregarding for the time the grids, the optical system, no matter how it is constructed, produces an image of the extended source S. Neglecting absorption and reflexion losses, this image will be of the same intrinsic brightness as the source S itself. The lens L_2 may thus be looked upon simply as an aperture through which the image can be seen. As long as the image appears to fill the aperture of L_2, when viewed from the receiver—that is, as long as S is not too far removed from the focus of the projecting system as a whole, the correction of this system for chromatic and other aberrations is not of the least importance, and makes no difference in the efficiency of the apparatus. In fact, the aperture of the lens L_2 may for all practical purposes be regarded as itself a luminous source of light. Coming, now, to the receiving system, shown in Fig. 3, the lens L_3 produces an image of the luminous aperture L_2 near its principal focus. All the useful light lies within a cone bounded by the aperture L_3 and the image of the aperture L_2 formed by the lens L_3. It is evident that the efficiency of the apparatus is increased by enlarging the apertures of both the projecting and the receiving systems (involving, in the latter case, an increasing focal length) up to the point where the image of L_2 in L_3 just covers the sensitive area of the selenium cell. These considerations show that the colour correction of the projecting system is not of importance, but that the

correction of the receiving system becomes important when the image of the aperture of the projecting system approaches its maximum useful size. There appears, however, to be very little justification for indulging in very expensive lenses of large aperture in this particular application of optical instruments. The conditions are so totally different from those under which optical instruments are normally employed that the limitations which are such an inevitable barrier to the optician can be evaded. Instead of employing a relay to amplify the current variation in the microphone circuit, with the consequent objectionable grating sound that has been mentioned, it would appear preferable to use a number of receiving sets, each consisting of an optical system and a selenium cell, and to connect these in parallel. There would be no need to make any special selection of lenses, as the high velocity of light will prevent the perception of any departure from perfect agreement in phase. The duplication of the projecting apparatus with the diaphragm of the trumpet as a common element, though not by any means impossible, presents some difficulties. Reverting to the projection system, there is little doubt that if grids are employed they must be placed approximately in the positions shown in Fig. 1. If C is the centre of curvature of the mirror A the grids should lie on the sphere, having AC as a diameter. The mirror A, and preferably also the source S, should lie within the focus of the projection system, so that their images are virtual. This will prevent the occurrence of bad spots in the projected field due to blemishes on the surface of the mirror. There would probably be decided advantages in dispensing with the grids entirely and using some other means for varying the intensity of the light. One consequence of the use of the grid is an initial reduction in the intensity of the light of 50 per cent. The difficulty is that the variation in intensity must be accomplished by very minute movements of a small piece of the apparatus. It follows that this portion of the apparatus must be situated near an image of the light source, and since this is roughly conjugate to a distant point of the external field no mechanism of the comb type can be admitted. It appears, however, possible that a glass absorbing wedge with a very sharp rate of increase of absorption with distance would be admissible. This would preferably be arranged to move in a vertical plane with its edge horizontal, so that the apparatus could be set up accurately if a spirit level were attached to the base, the adjustment in a horizontal plane being carried out as at present. A stationary inverted wedge of similar gradation could be employed to give a uniform field. The mirror A would in this case be removed, and the whole apparatus moved into line. By removing the wedges and the arc and substituting an eyepiece near S the apparatus would be converted into an erecting telescope, and could be used for sighting with great accuracy when long ranges were in question. With such an arrangement the special difficulties consequent on the use of the grid disappear, and the projecting system can be made much more compact. There is no reason why in this part of the apparatus lenses should not be employed having very large apertures with short focal lengths. There is little doubt that the author is correct in preferring to use lenses in place of mirrors.

Mr. A. P. Trotter described some experiments which he had been invited by the Admiralty through Dr. Eccles to carry out. He did not know who the inventor was, or if more successful results had been obtained elsewhere. His own results were not worthy of more than a place in the discussion on Dr. Rankine's Paper. Almost all the apparatus had been provided by Dr. Eccles. Small lamps having tungsten filaments about 5 mm. in length were filled with hydrogen or with mixtures of hydrogen and nitrogen. The best were filled with hydrogen at pressures from 200 mm. to 600 mm. below atmospheric. A circuit was made through a microphone and a lamp, either direct or through a transformer, and the filament responded to the undulations sufficiently to reproduce speech when the light was concentrated on the selenium cell. The utmost distance that he could transmit speech with

lenses 76 mm. (3 in.) diameter was about 2 metres. The cells were about 8 mm. square.

Dr. Eccles said that the method of light telephony just alluded to was an invention of his own, which he had hoped to describe at a future date along with two or three other novel methods. The idea underlying the method alluded to is to obtain a glow lamp with so fine a filament that its temperature and light emission could follow acoustically produced alterations of filament current. The best telephonic results were obtained when the lamp bulbs were filled with hydrogen. After the method had been shown feasible for speech frequencies, some of the early test lamps were sent to Mr. Trotter, and he was asked to measure, if possible, the actual fraction of a second taken by the filament to rise to and fall from its maximum brilliancy after a current was switched full on or right off. This he accomplished very successfully. Regarding Mr. T. Smith's suggested use of a black glass wedge moved by the voice to produce modulation of the flux from a steady source, the speaker said he had used and found excellent a similar and even simpler method, in which a sewing needle fixed to the sound box of a phonograph had its shadow projected by aid of an obvious optical system upon the distant selenium; the acoustic vibrations of the needle caused a magnified motion of the shadow which covered and uncovered the selenium correspondingly, and gave as the result very perfect articulation in a telephone receiver in series with the selenium.

Mr. F. E. Smith asked to what extent the cells were screened, as the sensitivity varied greatly with the total illumination of the cells. He thought that the method might be of great use in the development of speaking and singing cinematograph pictures, as the record could be in the form of a second film working beside the other, producing variations in a second beam of light in exact synchronism with the picture. A receiving system as in Dr. Rankine's arrangement would convert these fluctuations into speech.

Mr. P. R. Coursey suggested the use of a polarised beam of light modified in intensity by electromagnetic rotation.

The Author, in reply to Mr. Paterson's questions, pointed out that the selenium cells used are not sufficiently constant for the measurement of light, but such constancy is not demanded of them in the present case. The sensitive area is about $\frac{1}{4}$ in. square, and the light received is purposely spread over this area, and not brought to a small focus. No precise measurements have been made, but it is certain that the sensitivity to slow fluctuations is much greater than for rapid changes, and it is doubtful whether more than one-thousandth of the full sensitivity is released at speech frequencies. My experiments are in agreement with Mr. F. E. Smith's comment that screening from extraneous light is advantageous—this is one reason why a lens receiver is preferable to a mirror receiver. His suggestion regarding synchronism in cinematography is very interesting. I have always contemplated obtaining photographic records of the intensity of the beam by means of a moving film camera, with a view to reproduction by selenium, but this possible application to keeping sounds and pictures in time is new to me.

I find it rather difficult to follow Mr. T. Smith's criticisms of the optical system. I would have preferred if he had shown in what respect my calculation of the advantage accruing from achromatism breaks down. Mr. Smith is an expert on optics, but I think in this case he is wrong. I see no escape from the argument that a definite quantity of light of all colours, which falls on the projecting lens is confined to a certain area at any given distance if the lens is properly corrected, but is spread over a larger area in the absence of correction. The amount of light received per unit area is consequently less in the latter case. Mr. Smith speaks of *the focus* of the projecting system. But the system has an infinite number of foci for polychromatic light, and, in terms of his argument, if S is at the focus for violet light it is *too far removed* from the focus for red light for his deduction to

remain valid. In addition to this I may call attention to three practical points: (i.) That the best results I have obtained have undoubtedly been with an astronomical mirror as the projector; (ii.) that the edges of the beam projected through an uncorrected lens are actually found to be coloured; and (iii.) that Prof. R. W. Wood, in a Paper recently read before this Society, described a chromatic lens which he had used for long-distance signalling, and which, presumably, he had found superior to uncorrected ones. The reason why corrected lenses are not so important in the receiver is that there is very considerable latitude in the position of the selenium cell. It is easy to arrange that the whole of the light, including the coloured edge, falls on the sensitive area of the selenium. With regard to the suggested use of a multiple receiver, this is, of course, possible, but the same effect is obtained by increasing appropriately the aperture of a single receiver. I am afraid I cannot agree with Mr. Smith's suggestion that it might be a decided advantage to dispense with the grids entirely, particularly if a system of two light-absorbing wedges, as suggested by him, is the alternative. The present method was devised in order to escape the great difficulty of operating any form of shutter or its equivalent by means of the minute movements of the speech in receiving diaphragm. Light absorbing wedges would surely, in effect, *diminish*, instead of increase, these movements in the ratio of the tangent of the angle of the wedge. It is perfectly true, also, that in the present system 50 per cent. of the original light is lost, but it is unlikely that a pair of wedges could be made to secure any improvement in this respect. Prof. Eccles has told us that he has used this system, but I doubt whether he would claim that it is as efficient as the device which is the subject of the present Paper.

The Use of the Triode Valve in Maintaining the Vibration of a Tuning Fork. *By* Prof. W. H. Eccles.

A tuning fork was exhibited sustained in vibration by means of a triode valve instead of by means of contacts. In the form described, two electromagnets act upon the prongs, and the windings of one magnet are in the grid circuit; those of the other magnet are in the plate circuit of the tube. When the fork is in motion the E.M.F. induced in the grid magnet controls the current flowing in the plate circuit and its magnet, with the result that the motion is sustained.

By increasing the distances between the poles of each magnet and the prong confronting it, the apparatus may be adjusted so that the fork is just not maintained in oscillation. In this condition the arrival of a train of sound waves of exactly the right frequency to produce resonance causes the fork to start into vibration. The instrument is then a tuned relay. A large fraction of a minute is required in some adjustments to provoke the full response.

DISCUSSION.

Mr. F. E. Smith said he had at one time been working on somewhat similar lines to Dr. Eccles, with a rather different object—viz., the accurate measurement of a very small time interval. He used first a purely electrical arrangement, and found that a very constant frequency (about 1,000 per second) could be obtained if the electrical conditions were right. For certain reasons, however, it was thought desirable to employ a tuning fork, and one of those had been borrowed from Prof. Eccles. He had found it essential to put a condenser across the inductance of one of the magnets to get approximate tuning in the electrical circuit, otherwise the fork would not respond readily. In his arrangement the currents induced by the vibration of the fork produced oscillations of the filament of an Einthoven galvanometer, the deflections of which were photographically recorded.

XIX. *A Form of Knudsen's Vacuum Manometer.* *By* Lewis F. Richardson.

Received April 11, 1919.

In 1911 the author was in need of a vacuum gauge for measuring pressures of the order of 1 dyne cm.$^{-2}$, or less, in electric lamp bulbs for the Sunbeam Lamp Co. The McLeod gauge, ordinarily in use in the factory, was unsatisfactory for research purposes, because it does not measure the pressure of condensible gases such as the vapours of water, oil or mercury. Sir Joseph Thomson very kindly considered the question and suggested that attention should be directed to the Knudsen Manometer, which is free from this defect.

From the Kinetic Theory of Gases, Knudsen ("Ann. der Physik," XXXII., p. 812) deduced that the pressure of gas in equilibrium in a closed apparatus varies from point to point as the square root of the absolute temperature of the gas, when the dimensions of the apparatus are very small compared with the mean free path of the molecules. This proposition will be referred to as . . . . (1)

Smoluchowski ("Ann. der Physik.," XXXIV., 182) shows that Knudsen's formula requires that the moving plate should be close to fixed plates on *both* sides.

Without going into the general theory, we may notice the peculiarities of some typical cases. Consider a gas confined between two indefinitely large plane parallel plates at uniform but different temperatures, and suppose the gas so rarefied that collisions occur with the plates only. The motion of each molecule is a series of journeys in straight lines between the plates. On each journey the molecule gives out on stopping exactly the translatory momentum in a direction normal to the plates which it received on starting. So the pressures on the two plates must be equal, however widely their temperatures may differ. At first sight this might appear to contradict Knudsen's statement (1), but it does really not do so, for it will be shown that the temperature of the gas is perfectly uniform. The kinetic energy of a particle which last collided with the hot solid is presumably greater on the average than that of one coming from the cold solid. But the temperature, measured by the mean kinetic energy, depends on the relative numbers of these two classes of molecules in the element of volume. Denote this ratio by G. From the symmetry it

follows that G is uniform throughout the gas, and so the temperature is also uniform. The only way in which we can set up an inequality of temperature in the gas is by disturbing the symmetry, as, for instance, by confining the hotter area on the plates to a small patch on one of them. In the neighbourhood of the patch, G will differ from its value in the remote parts of the region.

In the present instrument the temperature is nearly uniform over each plate, but the symmetry is disturbed by the insertion between them of a small disc, which may be said to separate off a pair of compartments one on either side of it, communicating round its edge. Suppose, for simplicity, that the disc is midway between the fixed plates and parallel to them. The rates at which molecules are shot into the two compartments from the distant parts of the apparatus must be approximately equal. To the same approximation in the steady state the particles must be shot out of the two compartments at equal rates. But the rate of loss from either compartment must be proportional to the density and to the mean velocity in that compartment, as in the effusion experiments of Graham and Osborne Reynolds (Jeans, "Dynamical Theory of Gases," 2nd edition, § § 168, 170). Hence, when we compare the compartments with one another, the square roots of the absolute temperatures vary directly as the densities, and by Boyle's law inversely as the pressures, as in (1) above.

Knudsen constructed an instrument operated by these local differences of pressure. However, the instrument, as originally described by him, could only be kept in operation for a few minutes at a time, as it depended on an inequality of temperature which obliterated itself soon after it had been set up.

The following instrument was designed in consultation with Mr. R. M. Abraham, of Messrs. C. F. Casella & Co. It was made by that firm in 1912 and 1913.

The hot and cold plates were of glass and about 10 cm. in diameter. They were separated by 0·54 cm. by a glass ring. One of the plates had a hole drilled in it for the insertion of the connection to the vacuum pump. The joints were made tight by the special soft sealing wax in use at Owen's College, Manchester.

The flat glass box, so formed, was placed between two massive copper slabs each 1·15 cm. thick.

A difference of temperature of about 10°C. could be maintained indefinitely between the glass plates by warming one of

the copper slabs by means of a small gas flame, or by cooling it with ice. To eliminate electrostatic forces the inner faces of the glass plates were platinized and were put in electric connection by a wire spring which pressed against both.

The moving system is shown in Fig. 1. Inside the glass box a disc of mica of 3·5 cm. diameter was free to approach or recede from the hotter glass plate by rotation about an axis in the plane of the mica, but some distance beyond its edge. This axis was formed from two pieces of tungsten wire, one above and one below in the same straight line.

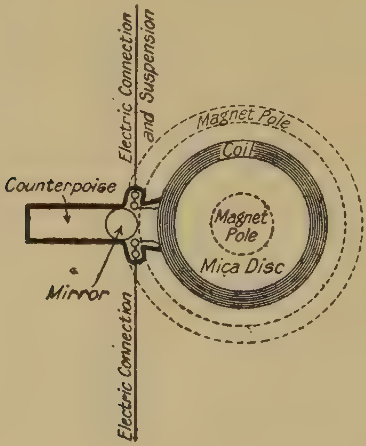


FIG. 1.

The force of the molecular bombardment tending to drive the moving disc away from the hotter glass plate, was balanced by an electromagnetic force. This force was produced by an electric current which flowed in a coil round the edge of the moving disc, and which was acted upon by a magnetic field directed along the radii of the circular coil. The poles of the magnet are shown in Fig. 1; they lay one on either side of the glass box. The object of this device was that the electromagnetic effect on the suspended system should be, not the usual couple, but a single force acting at the centre of the moving disc and at right angles to it.

The angular position of the moving system was observed by the light reflected from a mirror which it carried. The angle was brought exactly to zero by the current. The mica vane was kept approximately midway between the glass plates, but

its exact position in this translatory sense could not be observed, and, on Knudsen's theory, did not matter.

A pair of thermo-junctions of copper-eureka were nearly in contact with the outer sides of the glass box opposite the centre of area of the moving system. The E.M.F. given by this circuit was balanced on a potentiometer wire, through which flowed the current in the moving coil.

A defect of the instrument as constructed was a considerable twist in the suspension wire, which required a current J_0 to balance it, when both plates were at the same temperature. When a temperature-difference had been established a different current J_r was required. Thus, $J_r - J_0$ was proportional to the mechanical force produced by the gas. If we denote by r the resistance of the potentiometer wire required to make a balance with the thermo-junctions, then $r \,.\, J_r$ was proportional to the difference of temperature $(T_1 - T_2)$ between the affixed plates. So that the mechanical force per temperature difference was proportional to

$$\frac{1}{r}\left(1-\frac{J_0}{J_r}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Next, Knudsen's theory can be adapted to present circumstances as follows: The mechanical force per area is the difference of the gas pressures P_1 and P_2 on the two sides of the moving disc. The temperatures of the gas on the two sides may be taken to be $M + \frac{1}{4}\Delta$ and $M - \frac{1}{4}\Delta$, where M is the mean of T_1 and T_2 and Δ is their difference $T_1 - T_2$.

Now, if the gas in the instrument is in communication with a McLeod gauge which has a temperature T_3 and which registers a pressure P_3, then from Knudsen's formula (1) we shall have for the pressures P_1 and P_2 on the two sides of the moving disc:

$$\frac{P_1}{P_3} = \left\{\frac{M + \frac{1}{4}\Delta}{T_3}\right\}^{\frac{1}{2}}; \quad \frac{P_2}{P_3} = \left\{\frac{M - \frac{1}{4}\Delta}{T_3}\right\}^{\frac{1}{2}}, \quad . \quad . \quad . \quad (3)$$

whence, on expanding by the Binomial theorem,

$$\frac{P_1 - P_2}{P_3} = \frac{\Delta}{4\sqrt{MT_3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

if terms in Δ^3 and higher odd powers be neglected.

So $$\frac{4\sqrt{MT_3}\,.\,(P_1 - P_2)}{\Delta} = P_3. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Now, $(P_1-P_2)/\Delta$ is the mechanical force per area per difference of temperature, and hence by (2)—

$$C\frac{\sqrt{MT_3}}{r}\left(1-\frac{J_0}{J_r}\right)=P_3, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where C is a permanent instrumental constant to be determined by experiment. Note that this formula appears to assume that the McLeod gauge and the connecting tube were both small compared with the mean free path. That condition was not satisfied. If the initial twist were zero, and if the temperature T_3 of the McLeod gauge were equal to the mean temperature M of the Knudsen apparatus, then this formula would take the very simple form,

$$\frac{P}{T}=\frac{C}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where T is this temperature; that is to say, the *number of molecules per unit volume would be inversely proportional to the resistance of the potentiometer wire.*

Facilities for testing the apparatus were kindly afforded by Sir Ernest Rutherford at Owen's College.

The apparatus was exhausted by an oil pump and then by charcoal in liquid air, and was left in that state overnight. It was then again exhausted by liquid air twice; thus, it is probable that oil and water vapour were thoroughly removed. On the second occasion the connection to the charcoal tube was left open for half an hour after the McLeod gauge had begun to register a pressure less than 0·1 dyne/cm.^{2}. At the end of this time a reading of both gauges was taken. Next, the charcoal tube was shut off, so that the pressure gradually rose owing to the small leak in the apparatus. During this stage the other readings were taken. The contents were presumably mercury-vapour and air.

Fig. 2 shows the relation obtained in this way between the pressure P_3 by the McLeod gauge and

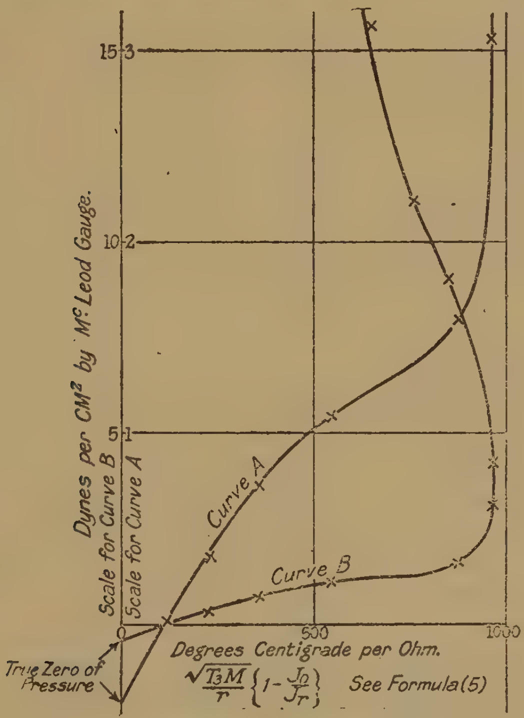
$$\sqrt{T_3M}\,.\,\frac{1}{r}\left(1-\frac{J_0}{J_r}\right)$$

from equation (5). The two curves represent the same function on different scales.

By extrapolation of curve A it is seen that the zero error of the McLeod gauge appears on this occasion to be about 0·4 dyne cm.$^{-2}$, which would correspond to half the maximum

pressure of mercury vapour at the temperature of the McLeod gauge, namely, 0·8 dyne cm.$^{-2}$ at, say, 13°C.

If we then take the true pressure to be given by the addition of 0·4 dyne cm.$^{-2}$ to the pressure obtained from the McLeod gauge, we may say that the useful range of this instrument of Knudsen's type extended up to 2·0 dynes cm.$^{-2}$. The mean temperature of the glass box was about 300°A. On this basis mean free paths have been computed from the data given for



PRESSURE BY KNUDSEN GAUGE.

FIG. 2.

air by Jeans (" Dyn. Theory of Gases," 2nd ed., p. 341). The top of the useful range was reached at 2·0 dynes cm.$^{-2}$ when the mean free path was six times the distance of 0·54 cm. between the inner faces of the glass box.

Curve *B*, drawn on a reduced scale, shows that the mechanical force per temperature-difference attained a maximum when the corrected pressure was 4·0 dynes cm.$^{-2}$, so that the

mean free path was 3·0 times the distance between the fixed plates. The maximum is, of course, outside the range of the formulæ (1) to (6), which only apply to the initial linear part.

The neglected terms in (3a) do not amount to 1/5,000 of the term which is retained, when Δ=17°C., as it was during the test, so that they cannot account for the curvature of the graph.

The chief difficulty experienced in use was that the electro magnetic force affected the period of oscillation of the coil, and in some circumstances rendered it unstable. Instability could probably be got rid of by new pole-pieces for the magnet, symmetrically designed to give as uniform a field as possible.

The vacuum-tightness of the apparatus may be measured inversely as a leak expressed as the rate of rise of pressure multiplied by the volume of all the connected vacuous cavities. It was found to be 0·2 dyne cm.$^{-2}$, sec.$^{-1}$, cm.^{3}. This would comprise also any evolution of occluded gases.

The resistance of the moving coil and its leading-in wires was 50 to 60 ohms. The current through it to balance the maximum force of the gas was about 1/700 ampere-hour per 10°C. difference between the plates, so that the electric heating of the coil was negligible.

Work upon apparatus had to be abandoned in 1913 owing to the pressure of other operations. Since then a vacuum-manometer depending on viscosity has been brought out by Dr. P. E. Shaw, and investigated theoretically by Mr. F. J. W. Whipple. A Paper on the Knudsen manometer, by J. W. Woodrow, has appeared in the "Physical Review" for December, 1914.

Summary.

The McLeod gauge has a false zero of pressure if condensible vapours are present. The Knudsen instrument is free from this defect, but has a very limited range. It operates on the principle that when the molecules collide only with the solid parts of the apparatus, then they knock a free vane away from a hotter towards a colder surface. The instrument as originally described by Knudsen could only be kept in operation for a few minutes at a time. The action of the present instrument could be maintained indefinitely. Its range extends up to 2·0 dynes cm.$^{-2}$. The force of the molecular bombardment is balanced by the effect of a magnetic field of special form, acting upon an electric current attached to the vane, and

the temperature difference is measured by a thermo-junction, the E.M.F. of which is balanced against the same current in a potentiometer. The instrument was constructed by Messrs. C. F. Casella & Co.

DISCUSSION.

Mr. R. S. WHIPPLE said he was not quite clear as to how the plate moved.

Mr. C. C. PATERSON asked if the size of the tube entering the manometer had much effect on the results.

Dr. D. OWEN asked if it would not be better to measure the temperature difference between the glass plates instead of the copper plates outside. As it was, the instrument had to be calibrated empirically, and could not be used to give a check on Knudsen's formulæ. With regard to curve 2, the author said that the mechanical force per unit temperature difference was a maximum at 4 dynes per square centimetre, and then stated that this lies outside the range of the formulæ. What, then, was the meaning of this part of the curve ? What was the lowest pressure which the author was able to measure ? If J_0' was nearly equal to J_r, the method evidently became inaccurate. Could not J_0 be eliminated ?

The AUTHOR, in reply, said that the plate moved at right angles to its own plane. The length of leading-in tube was about a metre, and its diameter about 1 cm. He did not remember the precise figures. The difference of temperature of the glass plates could easily be measured directly ; but it was easy to show that, under steady conditions, the temperature of the copper plate was a very fair measure of that of the glass. As regards a check on Knudsen's theory, there was at least a partial check, inasmuch as the first parts of the curve were straight. The linear law was not assumed, otherwise the diagram would consist of two straight lines. The lowest pressure he had reached was 0·4 dyne per square centimetre, owing to the vapour pressure of mercury in the apparatus. It would be useful to eliminate J_0 by taking extra trouble with the suspension wire ; but it did not introduce inaccuracy, as it simply had to be measured.

XX. *On Theories of Thermal Transpiration.* *By* GILBERT D. WEST, *M.Sc* (*Lond.*).

RECEIVED MAY 8, 1919.

IN connection with an experimental research arising out of measurements of the pressure of light,* the author has had occasion to consider the various theories that have been advanced to explain the phenomena of thermal transpiration. Of those considered, that formulated by Sutherland in 1896† has been most helpful. It is the object of the present Paper

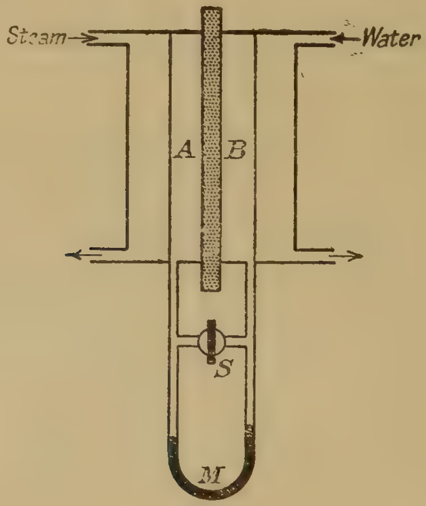


FIG. 1.

to indicate the relation of this theory to other theories, to show how it is capable of explaining work already done, and to put it into a form that will be of use in the further research it is hoped shortly to publish.

It is to Osborne Reynolds that we owe the discovery of thermal transpiration.‡ Reynolds used an apparatus somewhat similar to that shown in Fig. 1. The apparatus consists essentially of two chambers *A* and *B*, separated by a plate of porous material such as meerschaum. Means are provided for

* "Proc." Phys. Soc., XXVIII., p. 259, 1916.
† "Phil. Mag.," XLII., p. 373, 1896.
‡ Phil. "Trans.," CLXX., p. 727, 1879.

maintaining the chambers at different temperatures. M is a mercury manometer, and S is a stopcock.

On opening the latter, the pressures in A and B are equalised. When, however, it is closed again, the pressure in A gradually rises, as the result of the passage of gas from B to A through the meerschaum. When the mean free path of the gas molecules is large compared with the size of the pores, it is found that the final pressures in the chambers A and B are proportional to the square root of their absolute temperatures, T_A and T_B. It should be noted that the nature of the gas is unimportant.

The usual explanation of this result is somewhat as follows. Suppose the porous plate replaced by a non-conducting lamina with a single perforation, small compared to the mean free path of the gas molecules. Let N_A and V_A represent respectively the number and root-mean-square velocity of the molecules in the compartment A, and likewise let N_B and V_B represent similar magnitudes in regard to the compartment B. If all the molecules be divided into Joule's six conventional sets, moving parallel and perpendicular to the lamina, we shall have $\frac{1}{6}N_A V_A$ and $\frac{1}{6}N_B V_B$ impacts per square centimetre respectively on each side of the lamina. When equilibrium is reached the numbers of molecules passing each way through the orifice must be the same, and hence

$$\tfrac{1}{6}N_A V_A = \tfrac{1}{6}N_B V_B.$$

Thus, if m be the mass of a molecule, the ratio of the pressure in A and B is given by

$$\frac{P_A}{P_B} = \frac{\frac{1}{3}N_A m V_A^2}{\frac{1}{3}N_B m V_B^2} = \frac{V_A}{V_B} = \sqrt{\frac{T_A}{T_B}}.$$

There is no difficulty in extending this calculation to the case where the orifice is replaced by a fine bore tube, along which a temperature gradient is maintained, and the step to a porous plate is then simple.

A straightforward explanation can thus be given of the experimental results at low pressures. When, however, with rise of pressure, the mean free path of the molecules becomes comparable to, or less than the size of the pores, difficulties arise, and the simple theory previously given, no longer holds.

An elaborate investigation of the phenomena at such pressures was given by Osborne Reynolds, but it was presented in a form so abstruse that it is doubtful to what extent it cleared the minds of most physicists. Sutherland, for instance, in

his Paper* remarks that "unfortunately the mathematical form of Reynolds' theory is wearily cumbersome; one gathers that Maxwell found it distasteful, and Fitzgerald ("Phil. Mag.," (5) XI.) describes it as inelegant and unnecessarily elaborate . . . but what appears to me to be the fatal objection to Reynolds' mathematical method, is that it takes the mind away from definite physical concepts of the actual operation of the causes of thermal transpiration and radiometer motion." He goes on to say that the object of his own Paper "is to construct a theory . . . that will fall into line with the current kinetic theory of gases, and keep the physics of the phenomena to the fore."

Sutherland's Paper came, however, at a time when interest in thermal transpiration and cognate phenomena had died down, and it thus escaped sufficient attention. Sir Joseph Larmor in his article on "Radiometer" in the "Encyclopædia Britannica," does not even mention the Paper, and the Danish physicist, Knudsen, in a Paper† on thermal transpiration in its relation to the equilibrium conditions in a gas, does not mention the Paper, and moreover goes over a certain amount of ground that Sutherland had previously traversed.

From the point of view of the author's further research, both Sutherland's and Knudsen's Papers are of considerable importance, and, although both theories have a somewhat similar basis, the methods of development are so different that it has been thought desirable to show how, by a simple, though perhaps rather approximate, calculation based chiefly on Sutherland's methods, we can arrive at and extend Knudsen's results. More especially is this desirable, as the latter are supported by a number of carefully planned experiments.

Consider (as does Knudsen) the case of a tube along which a temperature gradient dT/dx is maintained. It is necessary to suppose this gradient to be small as otherwise λ the mean free path of the molecules may vary somewhat from point to point, and the caclulation then becomes very difficult. It is further necessary to suppose that the gradient is constant over a distance several times λ, that the diameter of the tube is large compared to λ, and, lastly, that any gas currents that occur in the tube are small in magnitude, in order that the temperature may be considered constant over any cross-section.

* *Loc. cit.*, p. 374.

† "Ann. d. Phys.," XXXI., 1, pp. 205-229, 1910.

Let the tube PQ (Fig. 2) connect two infinite spaces, and fix attention on a small area ds of a cross-section AB. Unless ds is very near the wall of the tube, molecules can be supposed to arrive at this area from points on the surface of a sphere of radius λ, and they can be divided into two classes—those with a component velocity parallel to the axis of the tube and towards the left, and those with a similar component velocity towards the right. If all the molecules in any section be supposed to have the arithmetic mean velocity Ω, and if further, the angle θ (measured according to the ordinary convention) represent

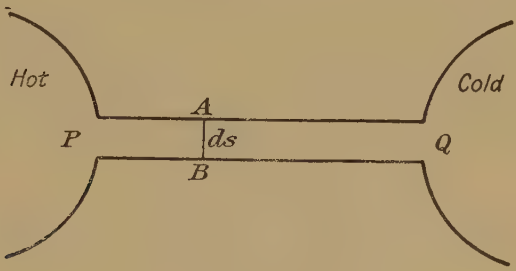


FIG. 2.

the inclination of the direction of motion of any molecule to the axis of the tube, the velocity of this molecule will be $\Omega+\frac{d\Omega}{dx}\lambda\cos\theta$, whether it comes from the left or right. Hence, if from a point O we draw lines representing the velocities of all the molecules that arrive at ds, from, say, the left, we shall have a diagram similar to Fig. 3, in which

$$OR=\Omega+\frac{d\Omega}{dx}\lambda, \text{ and } OS=\Omega.$$

Calculation is now much simplified if we assume all the molecules to have the same velocity $\Omega-\frac{1}{2}\frac{d\Omega}{dx}\lambda$. This approximation substitutes a hemisphere of radius $\Omega-\frac{1}{2}\frac{d\Omega}{dx}\lambda$ for the previous curved surface, and is hence more justifiable the smaller the value of $d\Omega/dx$. Likewise, we shall assume that if N be the number of molecules per unit volume at ds, the

number of molecules per unit volume possessing the velocity

$$\left(\Omega+\tfrac{1}{2}\lambda\frac{d\Omega}{dx}\right) \text{ is } \tfrac{1}{2}\left(N+\tfrac{1}{2}\lambda\frac{dN}{dx}\right).$$

On the basis of these assumptions, it thus appears that on the average the molecules at AB are made up of those arriving from distances $\frac{1}{2}\lambda$ to the left and right of AB respectively. Now it can be shown without difficulty* that the mean component velocity parallel to the axis of the tube of, say, all the molecules from the left possesssing the velocity $\left(\Omega+\frac{1}{2}\frac{d\Omega}{dx}\right)$ is given by

$$\tfrac{1}{2}\left(\Omega+\tfrac{1}{2}\lambda\frac{d\Omega}{dx}\right),$$

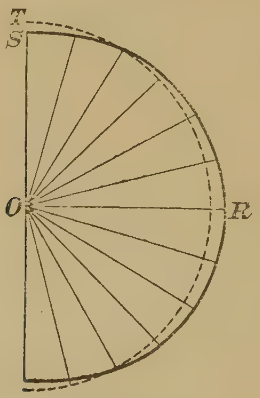


FIG. 3.

and hence the total mass flowing per second through unit area from left to right is

$$\tfrac{1}{4}\left(N+\tfrac{1}{2}\lambda\frac{dN}{dx}\right)\left(\Omega+\tfrac{1}{2}\lambda\frac{d\Omega}{dx}\right)m,$$

where m is the mass of a molecule, or approximately

$$\tfrac{1}{4}m\left\{N\Omega+\tfrac{1}{2}\lambda\Omega\frac{dN}{dx}+\tfrac{1}{2}\lambda N\frac{d\Omega}{dx}\right\},$$

since

$$\tfrac{1}{4}\lambda^2\frac{dN}{dx}\,\frac{d\Omega}{dx}$$

is a second order small quantity, and may be neglected.

* Meyer, "Kinetic Theory of Gases," 2nd ed., p. 83.

Similarly, we could show that the total mass flowing per second through unit area from right to left is given by

$$\tfrac{1}{4}m\left\{N\Omega-\tfrac{1}{2}\lambda\Omega\frac{dN}{dx}-\tfrac{1}{2}\lambda N\frac{d\Omega}{dx}\right\}.$$

Thus, the excess mass flowing from right to left through unit area in unit time is given by

$$-\tfrac{1}{4}m\left\{\lambda\Omega\frac{dN}{dx}+\lambda N\frac{d\Omega}{dx}\right\},$$

or,

$$-\tfrac{1}{4}mN\lambda\Omega\left\{\frac{1}{N}\frac{dN}{dx}+\frac{1}{\Omega}\frac{d\Omega}{dx}\right\}.$$

We have assumed that all the molecules have the same velocity Ω, but it is more accurate to assume them distributed according to Maxwell's law. In that case we must change $\frac{1}{4}$ to $3\pi/32$, and write the previous expression

$$-\frac{3\pi}{32}mN\lambda\Omega\left\{\frac{1}{N}\frac{dN}{dx}+\frac{1}{\Omega}\frac{d\Omega}{dx}\right\}.$$

Since, however, p the pressure of a gas, is given by

$$p=\frac{\pi}{8}Nm\Omega^2,$$

it follows that

$$\frac{dp}{dx}=\frac{\pi}{8}m\Omega^2\frac{dN}{dx}+\frac{2\pi}{8}Nm\Omega\frac{d\Omega}{dx},$$

$$=p\left\{\frac{1}{N}\frac{dN}{dx}+\frac{2}{\Omega}\frac{d\Omega}{dx}\right\}.$$

And further, since according to Maxwell η, the coefficient of viscosity of a gas, is given by $\eta=0\cdot31mN\lambda\Omega$, we may write the excess mass moving per second through a tube of radius R as

$$-\pi R^2\frac{3\pi}{32}\frac{\eta}{0\cdot31}\left\{\frac{1}{p}\frac{dp}{dx}-\frac{1}{\Omega}\frac{d\Omega}{dx}\right\},$$

or as

$$-2\cdot98R^2\eta\left\{\frac{1}{p}\frac{dp}{dx}-\frac{1}{2T}\frac{dT}{dx}\right\},$$

since the temperature T is proportional to Ω^2.

If the spaces at the ends of the tube are infinite this flow, which is uniform over the whole cross section, will continue as long as the temperature gradient is maintained. If, however, the spaces are limited, the flow will continue only until a sufficient pressure is developed on the hot side to

cause an equal flow of gas in the reverse direction. Such a counter-flow of gas, however, can only take place in conformity with the laws of flow in a capillary tube, whereby the velocity, starting from a maximum along the axis, must decrease progressively until the wall of the tube is reached. The result of the superposition of this flow on the uniform flow in the reverse direction will be to give us a gas current near the surface of the tube from the cold to the hot vessel, and a current in the reverse direction along the axis. Between the two there will be a surface of zero velocity. This superposition of two flows is the basis of Sutherland's method, and Fig. 4 is intended to give an idea of the distribution of velocities in this case.

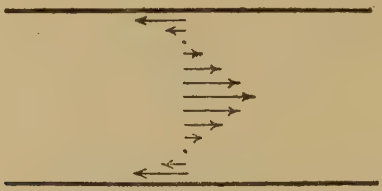


FIG. 4.

The simple formula of Poiseuille gives the mass of gas G discharged from a tube by a small pressure gradient as

$$G=\frac{\pi}{8}\frac{\rho_1 p R^4}{\eta}\frac{dp}{dx},$$

where ρ_1 is the density of the gas at T°C. and under a pressure of 1 dyne per square centimetre. The formula, however, takes no note of the "slip" that occurs at the surface of the tube. This latter is very important at the lower pressures, and hence a more elaborate expression is necessary. Various formulæ have been proposed,* and they differ in points of detail, but the following serves the present purpose—

$$G=\left\{\frac{\pi}{8}\frac{\rho_1 p}{\eta}R^4+\frac{4}{3}\sqrt{2\pi\rho_1}R^3\right\}\frac{dp}{dx}.$$

Equating, therefore, the masses discharged in the reverse directions, we have

$$-\frac{2\cdot 98R^2\eta}{p}\frac{dp}{dx}+\frac{2\cdot 98\eta}{2T}R^2\frac{dT}{dx}=\left\{\frac{\pi}{8}\frac{\rho_1 p}{\eta}R^4+3\cdot 34\sqrt{\rho_1}R^3\right\}\frac{dp}{dx}. \quad (1)$$

* Fisher, "Phys. Rev.," XXIX., p. 325, 1909.

Whence

$$\frac{dp}{dT}=\eta^2\Big/\{2\eta^2T/p+37{\cdot}1R\eta\sqrt{\rho_0T}+65{\cdot}1\,\rho_0pR^2\}, \quad . \quad . \quad (2)$$

where ρ_0 = mass of 1 cubic cm. of gas at 0°C. under 1 dyne/cm.^{2} pressure.

This formula has been deduced on the assumption that the pressure is of a value such that R is large compared to λ, and it will be remembered that we found the molecules could be considered to come from distances $\frac{1}{2}\lambda$ from either side of the section AB in Fig. 2. When the pressure is so low that λ is large compared to the diameter of the tube, this is not so. We may replace the $\frac{1}{2}$, however, by a quantity k, which will be very approximately constant, so long as λ is great.

Hence, instead of equation (1), we must now write

$$-\pi R^2\frac{3\pi}{32}mN\lambda\Omega\left\{\frac{2k}{p}\frac{dp}{dx}-\frac{2k}{T}\frac{dT}{dx}\right\}$$

$$=\left\{\frac{\pi}{8}\frac{\rho_1p}{\eta}R^4+\frac{4}{3}\sqrt{2\pi\rho_1}R^3\right\}\frac{dp}{dx}.$$

Since, however, λ is by supposition much greater than R, the equation reduces in the limit to an equation independent of k, namely,

$$\left\{\frac{1}{p}\frac{dp}{dx}-\frac{1}{2T}\frac{dT}{dx}\right\}=0,^*$$

which gives us

$$\frac{dp}{dT}=\frac{p}{2T}.$$

Now, this is the limit to which the previous formula (2) reduces when p is made very small. Hence, the formula 2 applies to both high and low pressures. In regard to medium pressures, we should in strictness have to introduce an appropriate value of k, but it appears that no great error is made by leaving the formula untouched. Hence, the equation

$$\frac{dp}{dT}=\eta^2\Big/\left\{\frac{2\eta^2T}{p}+37{\cdot}1R\eta\sqrt{\rho_0T}+65{\cdot}1\,\rho_0pR^2\right\}$$

may be said to apply approximately to all pressures. The variation of dp/dT with gas pressure for a tube of definite radius is shown in the accompanying curves. It will be seen

* Physically, this implies that, in the condition of equilibrium there is no flow—an assumption made in the simple explanation given at the beginning of the Paper.

that with decreasing pressure dp/dT rises, reaches a maximum and then falls.

It is now necessary to compare these results with those of Knudsen. An outline of his method is somewhat as follows :

A calculation is first made of the traction a small length of tube, along which a temperature gradient is maintained, would experience in the absence of gas currents. In a detailed calculation it is then shown how the traction is modified when the peripheral gas current from the cold vessel discharges as much as the axial current from the hot vessel. He equates the traction thus found to that calculated from the pressure gradient and the cross-section of the tube.

As a result he finds for high pressures, and for medium pressures approached from the high-pressure side,

$$\frac{dp}{dT}=\eta^2K\Big/\{43{\cdot}5R\eta\sqrt{\rho_0 T}+68{\cdot}4\rho_0 R^2 p\},$$

where K varies from 1 at high pressures to 4/3 at very low pressures. Likewise, for low pressures and for medium pressures approached from the low-pressure side, he finds that

$$\frac{dp}{dT}=\tfrac{3}{8}Kp\Big/\left(1+\frac{2R}{\lambda}\right)T,$$

which, when $K=4/3$ can be reduced to

$$\frac{dp}{dT}=p\eta\Big/\{2T\eta+32{\cdot}7pR\sqrt{\rho_0 T}\}.$$

When $K=1$, 32·7 is replaced by 43·5—*i.e.*, the value in the previous formula.

Except in so far as Knudsen's results are not combined in a single formula, they are of the same form as those obtained in this work, whilst the discrepancy of the coefficients may be explained chiefly by the variation of K. The principles laid down by Sutherland would hence appear to form an adequate basis for the more modern work of Knudsen.

The accompanying curves plotted on semi-logarithmic paper so as to secure a greater range, illustrate well the relationship of the various formulæ. At low pressures we see that both agree in making dp/dT, the pressure difference per degree difference of temperature, proportional to the pressure, and independent of the nature of the gas. Further, at high pressures dp/dT becomes inversely proportional to the pressure and dependent on the nature of the gas. Both these

results are in accord with the experiments of Osborne Reynolds, to which previous reference has been made.

For the purposes of the experimental verification of his theory Knudsen found it necessary to integrate some of his equations, in order that they might apply when large differences of temperature existed. His results calculated in this way were in good accord with those obtained by his experiments, and considerable confidence may therefore be felt in

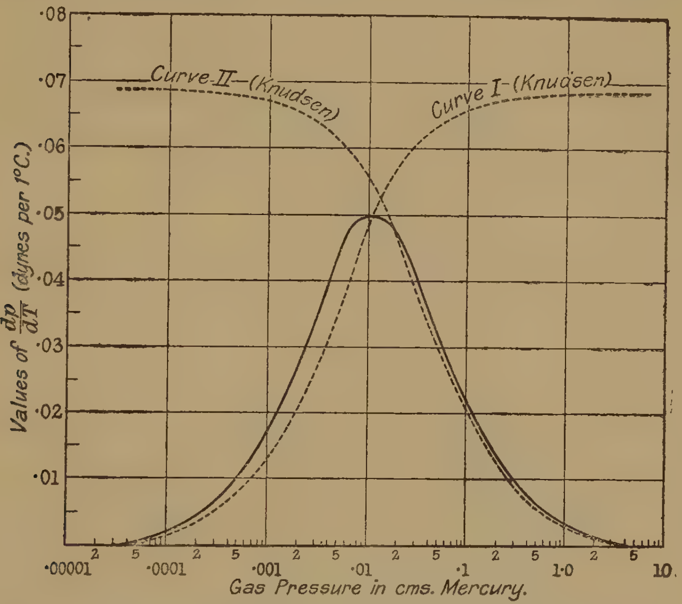


FIG. 5.—THERMAL TRANSPIRATION FORMULÆ FOR A CYLINDRICAL TUBE OF RADIUS 1 CM.

Full line curve given by $\dfrac{dp}{dT}=\eta^2\Big/\left\{\dfrac{2\eta^2 T}{p}+37{\cdot}1R\eta\sqrt{\rho_0 T}+65{\cdot}\rho_0 pR^2\right\}$.

Dotted line curve I. given by $\dfrac{dp}{dT}=\frac{3}{8}p\Big/\left(1+\dfrac{2R}{\lambda}\right)T$ (Knudsen).

Dotted line curve II. given by $\dfrac{dp}{dT}=\eta^2\Big/\left\{43{\cdot}5R\eta\sqrt{\rho_0 T}+6{\cdot}84\rho_0 R^2 p\right\}$ (Knudsen)

them. It is hence satisfactory to find that those developed in this Paper do not differ very materially from those of Knudsen, and for approximate calculations they are thus quite adequate.

Summarising, therefore, we may say that if two vessels containing gas at different temperatures are connected by a

capillary tube, the phenomena vary according to the relation of the mean free path of the molecules to the radius of the tube.

At very low pressures, gas flows from the cold vessel to the hot vessel until a sufficient pressure is developed to check it. If the difference in temperature be small, the pressure difference is proportional to the gas pressure in the two vessels, is independent of the nature of the gas, and is given in fact by

$$dp = \frac{p}{2T} dT.$$

With increasing pressure, the pressure difference rises less rapidly, and eventually reaches a maximum. It then begins to fall off and finally diminishes inversely as the pressure, approximately according to the equation,

$$dp = \eta^2 dT / 65{\cdot}1 \rho_0 p R^2.$$

In these latter stages a circulation of gas is maintained, and currents flow from cold to hot along the surface of the tube, and from hot to cold along the axis.

Summary.

The process of thermal transpiration, or the establishment of a pressure difference between two vessels connected by a capillary tube and at different temperatures, takes place at all gas pressures. The explanation of the phenomena at low pressures is well known and simple. When, however, the mean free path of the molecules is of the order of, or smaller than, the diameter of the tube, the simple explanation fails and a more elaborate hypothesis is necessary. Reference is made in the Paper to the work of Reynolds, Sutherland and Knudsen. The two latter investigators have proceeded on different lines, but it is shown how Sutherland's original method of treatment can be employed to calculate a formula which is applicable to all pressures, and which is in approximate agreement with the formulæ given by Knudsen for limited ranges of pressure. It is anticipated that the present Paper will be of considerable use in the author's further research.

DISCUSSION.

Mr. F. J. Whipple observed that the author had adopted the simplification of assuming all the molecules to have the mean velocity. Had he worked out the full treatment on the basis of the Maxwellian distribution of velocities? He stated that in order to bring the formula into accord with the Maxwellian distribution 1/4 had to be replaced by $3\pi/32$. Was it

easy to show this? He had not followed the author's explanation of the circulation set up in the tube. It was easy to see why the pressure-difference flow should be faster near the axis of the tube than at the walls; but the assumption that the temperature-difference flow should be uniform across the tube seemed somewhat arbitrary.

Dr. H. S. ALLEN was glad to hear the author's tribute to Sutherland. He had come across many cases of neglect of Sutherland's work. For example, the relation between the coefficient of expansion and the specific heat of an element usually attributed to Grüneisen, was first published by Sutherland, and there were many similar cases which could be quoted.

Prof. LEES congratulated the author on the simple way in which he had arrived at Knudsen's results.

The AUTHOR, in reply to Mr. WHIPPLE, said the numerical transformation referred to was simply made in conformity with the prevalent custom of replacing $1/4$ by $3\pi/32$ in such calculations. As regards the distribution of the temperature-difference flow, the value near the centre is obtained by integration over a hemisphere, while near the edges integration had to be performed over a smaller area—say half a hemisphere. The values obtained did not differ greatly at different parts of the cross-section.

XXI. *A Comparison of the Wave-form of the Telephone Current Produced by a Thermal Detector and by a Rectifier in Heterodyne Reception.* *By* BALTH. VAN DER POL, Jun., *D.Sc.* (*Utrecht*).

COMMUNICATED BY PROF. W. H. ECCLES.

RECEIVED MAY 9, 1919.

THE heterodyne method of receiving wireless signals is principally used for continuous wave reception. The well-known procedure is to superimpose on the sinusoidal antenna current due to the waves, another sinusoidal current generated locally. The wave-form of this complex current is then modified by some wave distorting detector in such a way that at least one harmonic will fall within the region of audibility, thus producing a sound in the telephone receiver.

The question naturally arises of what wave-form the resulting current will be.

A theoretical treatment obviously depends on the assumptions made underlying the action of the detector.

When a detector is used generating a potential difference at the telephone terminals at any time proportional to the square of the total antenna current the theory is very simple indeed. For, if $I_1 \cos \omega_1 t$ is the current due to the incoming waves and $-I_2 \cos \omega_2 t$ is the current generated locally, the potential difference at the terminals of the telephone receiver is obviously proportional to

$$(I_1 \cos \omega_1 t - I_2 \cos \omega_2 t)^2 = \tfrac{1}{2}(I_1^2 + I_2^2) + \tfrac{1}{2}(I_1^2 \cos 2\omega_1 t + I_2^2 \cos 2\omega_2 t) - I_1 I_2 \{\cos(\omega_1 + \omega_2)t + \cos(\omega_1 - \omega_2)t\}.$$

and the sinusoid of lowest frequency is seen to be

$$-I_1 I_2 \cos(\omega_1 - \omega_2)t.$$

If, on the other hand, a rectifier is used, *i.e.*, a system having a finite constant resistance for currents in one direction and an infinite resistance for currents in the other direction, the theory, if no approximations are to be used, becomes more involved. The telephone current in this case will be

$$\left.\begin{array}{l} I_1 \cos \omega_1 t - I_2 \cos \omega_2 t, \\ \text{with the condition that, when this func-} \\ \text{tion is negative, it is replaced by zero.} \end{array}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

The function (1) has zeros of a very complicated nature, and the Fourier analysis, which obviously depends to a large

extent on the values of these zeros, is in general a matter of great difficulty.

If, however, the amplitudes of both sinusoids are taken to be equal (equal heterodyne), the analysis becomes much simpler, and this case only will be considered here. Hence the Fourier analysis is required of

$$\left.\begin{array}{c}\cos\omega_1 t-\cos\omega_2 t,\\ \text{with the condition that, when this function is negative, it is replaced by zero.}\end{array}\right\} \quad . \quad . \quad . \quad (1a)$$

At the outset it must be remarked that $(1a)$ is only capable of being analysed in a Fourier series when ω_1 and ω_2 are commensurable.

Now, any function $f(t)$, subject to the condition that when it is negative it is replaced by zero, can be represented as

$$\tfrac{1}{2}\{|f(t)|+f(t)\}.$$

The function under consideration is therefore

$$\tfrac{1}{2}\{|\cos\omega_1 t-\cos\omega_2 t|+(\cos\omega_1 t-\cos\omega_2 t)\},$$

which we wish to express in the form

$$\begin{aligned}&b_0+b_1\cos ft+b_2\cos 2ft+\ .\ .\ .\\&+a_1\sin ft+a_2\sin 2ft+\ .\ .\ .\ .\end{aligned}$$

(where the meaning of f will be considered later).

The second term $\frac{1}{2}(\cos\omega_1 t-\cos\omega_2 t)$ only contributes to the value of b_{n1} and b_{n2}, where

$$n_1 f=\omega_1 \quad \text{and} \quad n_2 f=\omega_2,$$

and can, therefore, for the present be omitted.

The remaining part

$$\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|$$

is symmetrical with respect to $t=0$, therefore

$$a_1=a_2=.\ .\ .\ .\ .=0.$$

It can further be expressed as the product of two moduli, viz. :—

$$\begin{aligned}\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|&=|-\sin\frac{\omega_1+\omega_2}{2}t\,.\,\sin\frac{\omega_1-\omega_2}{2}t|\\&=|\sin\frac{\omega_1+\omega_2}{2}t|\,.\,|\sin\frac{\omega_1-\omega_2}{2}t|.\quad .\quad .\quad .\quad (2)\end{aligned}$$

Now the Fourier analysis of the modulus of $\sin pt$ is easily found* to be

$$\left.\begin{array}{l} |\sin pt| = \dfrac{2}{\pi}\left\{1-2\sum\limits_{l=1,2,3\ldots}^{\infty}\dfrac{1}{4l^2-1}\cos 2plt\right\}. \\ \text{In the same way we have} \\ |\cos pt| = \dfrac{2}{\pi}\left\{1-2\sum\limits_{l=1,2,3\ldots}^{\infty}\dfrac{(-1)^l}{4l^2-1}\cos 2plt\right\}. \end{array}\right\} \dagger \quad (3)$$

The function $\frac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|$ can therefore be written

$$\frac{4}{\pi^2}\left\{1-2\sum_{l=1,2,3\ldots}\frac{1}{4l^2-1}\cos(\omega_1+\omega_2)lt\right\}\left\{1-2\sum_{l'=1,2,3\ldots}\frac{1}{4l'^2-1}\cos(\omega_1-\omega_2)l't\right\}$$

$$=\frac{4}{\pi^2}\left\{1-2\sum_{l=1,2,3\ldots}\frac{1}{4l^2-1}\cos(\omega_1+\omega_2)lt,\right.$$

$$-2\sum_{l'=1,2,3\ldots}\frac{1}{4l'^2-1}\cos(\omega_1-\omega_2)l't,$$

$$\left.+4\sum_{l'=1,2,3\ldots}\sum_{l'=1,2,3\ldots}\frac{1}{4l^2-1}\,\frac{1}{4l'^2-1}\cos(\omega_1+\omega_2)lt\,.\cos(\omega_1-\omega_2)l't\right\}.$$

Each term of the last double series can be modified into

$$\frac{1}{2}\,\frac{1}{4l^2-1}\,\frac{1}{4l'^2-1}$$

$$[\cos\{(\omega_1+\omega_2)l+(\omega_1-\omega_2)l'\}t+\cos\{(\omega_1+\omega_2)l-(\omega_1-\omega_2)l'\}t],$$

so that we arrive at

$$\left.\begin{array}{l} \frac{1}{2}|\cos\omega_1 t-\cos\omega_2 t| = \\ \dfrac{4}{\pi^2}\left\{1-2\sum\limits_{l=1,2,3\ldots}\dfrac{1}{4l^2-1}\cos(\omega_1+\omega_2)lt\right. \\ \quad -2\sum\limits_{l'=1,2,3\ldots}\dfrac{1}{4l'^2-1}\cos(\omega_1-\omega_2)lt \\ \quad +2\sum\limits_{l=1,2,3\ldots}\sum\limits_{l'=1,2,3\ldots}\dfrac{1}{4l^2-1}\,\dfrac{1}{4l'^2-1}[\cos\{(\omega_1+\omega_2)l \\ \quad \left.+(\omega_1-\omega_2)l'\}t+\cos\{(\omega_1+\omega_2)l-(\omega_1-\omega_2)l'\}t]\right\} \end{array}\right\} \quad (4)$$

* See *e.g.* Riemann-Weber. Partielle Diff. Gleichungen I.

† These expressions at once give the analysis for the (*rectified* $\sin pt$)

$$=\tfrac{1}{2}\{|\sin pt|+\sin pt\}=\frac{1}{\pi}\left\{1-2\sum_l\frac{1}{4l^2-1}\cos 2plt+\frac{\pi}{2}\sin pt\right\},$$

and (*rectified* $\cos pt$)

$$=\tfrac{1}{2}\{|\cos pt|+\cos pt\}=\frac{1}{\pi}\left\{1-2\sum_l\frac{(-1)^l}{4l^2-1}\cos 2plt+\frac{\pi}{2}\cos pt\right\}.$$

The fundamental frequency of this function is $\frac{1}{2\pi}$ *times the greatest common factor (including fractions) of* $(\omega_1+\omega_2)$ *and* $(\omega_1-\omega_2)$, *so that we can write*

$$\omega_1+\omega_2=(p+q)f,$$
$$\omega_1-\omega_2=(p-q)f,$$

where $(p+q)$ *and* $(p-q)$ *are the smallest integers allowing* $(\omega_1+\omega_2)$ *and* $(\omega_1-\omega_2)$ *to be expressed in this form.* It is, further, not strictly necessary that f is at the same time the greatest common factor (in the sense referred to above) of ω_1 and ω_2, neither is it in general equal to $\omega_1-\omega_2$.

It is now only necessary to replace t by t'/f to get the required Fourier analysis

$$\left.\begin{aligned}&\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|=\\&\frac{4}{\pi^2}\Bigg\{1-2\sum_{l=1,2,3\ldots}\frac{1}{4l^2-1}\cos(p+q)lt'\\&\quad-2\sum_{l'=1,2,3\ldots}\frac{1}{4l'^2-1}\cos(p-q)l't'\\&\quad+2\sum_{l=1,2,3\ldots}\;\sum_{l'=1,2,3\ldots}\frac{1}{4l^2-1}\,\frac{1}{4l'^2-1}\\&[\cos\{(p+q)l+(p-q)l'\}t'+\cos\{(p+q)l-(p-q)l'\}t']\Bigg\}\end{aligned}\right\} \quad (5)$$

In order to find the amplitude of a certain harmonic the different terms in the series in (5) have to be carefully considered and collected. For instance, the direct component is made up of the first term together with all terms of which the factor of t' under the cosine signs is zero. They can only occur in the last double series, and those pairs of positive integral values of l and l' must be determined (excluding $l=l'=0$), for which either

$$(p+q)l+(p-q)l'=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or

$$(p+q)l-(p-q)l'=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where all variables are integers.

These being found, the corresponding expressions

$$2\cdot\frac{1}{4l^2-1}\cdot\frac{1}{4l'^2-1}$$

have to be calculated and the *exact* amplitude of the continuous component is therefore

$$b_0=\frac{4}{\pi^2}\left[1+2\sum_l\sum_{l'}\frac{1}{4l^2-1}\cdot\frac{1}{4l'^2-1}\right],$$

where in the sum those pairs of values of l and l' are to be taken which satisfy either (6) or (7).

Obviously if $p>q$, $p>0$ and $q>0$, (7) only can furnish us with those values.

In the same way the amplitude of the fundamental is given by those terms of the double series for which the coefficient of t' equals unity, *i.e.*, for which

$$(p+q)l+(p-q)l'=\pm 1 \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$(p+q)l-(p-q)l'=\pm 1 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

After the pairs of positive integral values of l and l' have been determined from the four equations (8) and (9), the amplitude of the fundamental is found to be

$$b_1=\frac{8}{\pi^2}\sum_l\sum_{l'}\frac{1}{4l^2-1}\cdot\frac{1}{4l'^2-1},$$

where again the summation is to be extended over all the pairs of values of l and l' which satisfy (8) and (9).

In general, the third term of (5) (second series), viz.,

$$-2\sum_{l'=1,2,3\ldots}\frac{1}{4l'^2-1}\cos(p-q)l't',$$

will *not* contribute to the fundamental, for it would then be necessary that a positive integer l' exists satisfying

$$(p-q)l'=\pm 1,$$

which is only possible when

$$p-q=1,$$

which is not the general case.

This second series, however, does contribute to the amplitude of the harmonic of order $p-q$. In fact we have

$$b_{p-q}=\frac{4}{\pi^2}\left\{-\frac{2}{3}+2\sum_l\sum_{l'}\frac{1}{4l^2-1}\cdot\frac{1}{4l'^2-1}\right\},$$

where the summation is to be extended over the pairs of positive integer values of l and l' satisfying the four equations

$$(p+q)l+(p-q)l'=\pm(p-q) \quad . \quad . \quad . \quad . \quad (10)$$

$$(p+q)l-(p-q)l'=\pm(p-q) \quad . \quad . \quad . \quad . \quad (11)$$

In general the second series in (5) contributes to the amplitude of the harmonics b_{p-q}, $b_{2(p-q)}$, $b_{3(p-q)}$, ..., &c., to the amounts respectively

$$2\cdot\frac{1}{4\cdot 1^2-1}=\frac{2}{3},$$

$$2\cdot\frac{1}{4\cdot 2^2-1}=\frac{2}{15},$$

$$2\cdot\frac{1}{4\cdot 3^2-1}=\frac{2}{35},$$

. . . . . ., &c.

In the same way the first series, viz.,

$$2\sum_{l}\frac{1}{4l^2-1}\cos(p+q)lt',$$

will contribute only to the amplitude of the harmonics

$$b_{p+q},\quad b_{2(p+q)},\quad b_{3(p+q)},\ .\ .\ .\ .\ .\ .\ \&c.,$$

to the amounts respectively again

$$\frac{2}{3},\quad \frac{2}{15},\quad \frac{2}{35},\ .\ .\ .\ .\ .\ .\ .\ \&c.$$

As $(p-q)$ and $(p+q)$ have, by definition, no common factor it will not be possible for the second series and the third to contribute to one and the same harmonic with the exception of $b_{n(p^2-q^2)}$ where n is an integer. However, in the way shown above, the third (double) series $\Sigma\Sigma$ will contribute to every harmonic, and this contribution has in all cases the form of a rapidly converging series,

$$2\sum_{l}\sum_{l'}\frac{1}{4l^2-1}\cdot\frac{1}{4l'^2-1},$$

where the values of l and l' over which the summation is to be extended have to be found from four Diophantine equations, similar to (8) (9) or (10) (11).

The analysis can further easily be extended to the cases of different initial phases from those considered above.

As regards the relative magnitudes of the amplitudes of the different harmonics these are wholly determined by the values of p and q. If $(p+q)$ is much greater than $(p-q)$, as is usually the case in heterodyne reception, the contributions of the double series to the amplitude of a certain harmonic are usually small in comparison with unity or (if they contribute at all)

with the single term furnished by either the first or second (single) series. We see, therefore, that the harmonics of order

$$(p-q),\ 2(p-q),\ 3(p-q),\ \ldots\ldots\ \&c.$$

and $$(p+q),\ 2(p+q),\ 3(p+q),\ \ldots\ldots\ \&c.$$

will have much greater amplitudes than the remaining harmonics. Again, in this case b_{p-q} will have a value very near that of b_{p+q},

$$b_{p-q} \sim b_{p+q} \sim \frac{-4}{\pi^2} \cdot \frac{2}{3}.$$

Again $$b_{2(p-q)} \sim b_{2(p+q)} \sim \frac{-4}{\pi^2} \cdot \frac{2}{15},\ \ldots\ldots\ \&c.$$

If the amplitudes of the different harmonics are represented graphically, we get a figure somewhat like Fig. 1*a*.

We see, therefore, that the amplitude of the harmonic of order $(p-q)$ is for practical cases much bigger than that of the fundamental, and it is likely that the ear will take the

FIG. 1*a*.

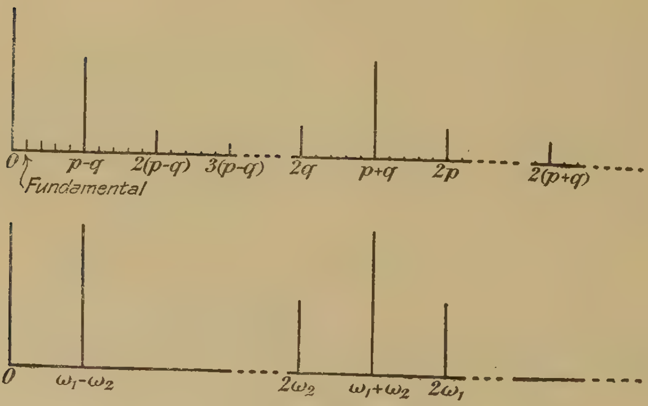


FIG. 1*b*.

harmonic of order $(p-q)$ as determining the pitch of the sound in the telephone receiver. Directly underneath (Fig. 1*b*) we give the amplitudes of the different harmonics that will be present on the assumption of a "thermal" detector, on such a scale that the direct current component in both cases is the same.

From the figure we see further the great theoretical advantage, as far as interference from other waves is concerned, of the thermal detector over the rectifier. While the rectifier gives strong harmonics of the order $2(p-q)$, $3(p-q)$, ..., &c., these fail in the sound produced by the thermal detector. The chance of interference by disturbing waves is, therefore, less when a thermal detector is used than with a rectifier.

In conclusion, we shall compare the mean square current produced by the rectifier (having a constant resistance R in one direction and an infinite resistance in the other direction) with the mean square current obtained with an ordinary ohmic resistance of the same magnitude R in both directions.

In the first case the current will be

$$\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|+\tfrac{1}{2}(\cos\omega_1 t-\cos\omega_2 t),$$

while in the second case it simply is

$$(\cos\omega_1 t-\cos\omega_2 t).$$

The square of the rectified current is

$$\tfrac{1}{4}\{|\cos\omega_1 t-\cos\omega_2 t|\}^2+\tfrac{1}{4}(\cos\omega_1 t-\cos\omega_2 t)^2+\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|\,.\,(\cos\omega_1 t-\cos\omega_2 t)=\tfrac{1}{2}(\cos\omega_1 t-\cos\omega_2 t)^2+\tfrac{1}{2}|\cos\omega_1 t-\cos\omega_2 t|\,.\,(\cos\omega_1 t-\cos\omega_2 t).$$

It follows that if we call I_0^2 the mean square of the unrectified current and I_r^2 the mean square of the rectified current we have

$$I_0^2=1,$$

$$I_r^2=\tfrac{1}{2}+\tfrac{1}{2}\frac{f}{2n}\int_0^{\frac{2n}{f}}(\cos\omega_1 t-\cos\omega_2 t)\,.\,|\cos\omega_1 t-\cos\omega_2 t|\,dt.$$

Now from the above analysis we find at once

$$b_{\omega 1}=\tfrac{1}{2}\frac{f}{2n}\int_0^{\frac{2n}{f}}\cos\omega_1 t\,.\,|\cos\omega_1 t-\cos\omega_2 t|\,dt,$$

and

$$b_{\omega 2}=\tfrac{1}{2}\frac{f}{2n}\int^{\frac{2n}{f}}\cos\omega_2 t\,.\,|\cos\omega_1 t-\cos\omega_2 t|\,dt,$$

where $b_{\omega 1}$ and $b_{\omega 2}$ can be determined in the way shown above.

Hence we finally obtain

$$I_r^2=\tfrac{1}{2}+b_{\omega 1}-b_{\omega 2},$$

as the *exact* value of the mean square of the rectified current.

A reversal of the detector in the circuit would give the mean square current

$$I_r'^2=\tfrac{1}{2}-b_{\omega 1}+b_{\omega 2}.$$

A *thermal instrument* reading mean square current would theoretically, therefore, give different readings in the two cases.

It follows from the above analysis that a *direct current instrument* would indicate rigorously the same current for both positions of the detector.

As $b_{\omega 1}$ and $b_{\omega 2}$, however, are usually small compared with unity, the readings of the thermal instrument for the two positions of the rectifier will differ very little, and, as was to be expected, we find for the *approximate* value for the mean square rectified current half that of the unrectified current, provided that the unidirectional resistance in the first case is equal to the ordinary resistance in the second.*

* I am indebted to Mr. F. J. W. Whipple for some comments on the unrevised proof.

XXII. *The Magnetic Properties of Varieties of Magnetite.*

By ERNEST WILSON *and* E. F. HERROUN.

RECEIVED MAY 19, 1919.

1. *Introductory.*

MAGNETITE is so widely distributed throughout the earth's crust, and is so interesting in many ways, that no apology is needed for describing experiments which throw further light upon the part played by this wonderful substance in influencing the magnetic properties of the rocks of which it is a constituent, or with which it comes in contact. Much work has already been done :—Abt* examined the retentivity of short and long bars and compared it with that of glass hard steel. Weiss† has made probably the most exhaustive experiments on magnetite crystals. He showed that the substance has a curve of magnetisation resembling that of iron, and that the susceptibility along the different crystallographic axes was different, although tending towards the same limiting value as the force increased. E. Holm ‡ has shown that in the case of Swedish magnetite the susceptibility varies with the magnetising force. The transformation temperature of magnetite has been examined by Barton and Williams, § Curie || and Allan¶.

The primary object of the present communication is to examine the magnetic properties of the various forms in which magnetite is found, ranging from the crystal itself to such substances as magnetite-calcite. Specimens from different parts of the world have been chosen, and the authors have been fortunate in that they have had placed at their disposal the collection of magnetite of the Geological Survey and Museum, numbering about 30 varieties. Prof. H. Louis has supplied certain specimens which he himself has collected, and in addition there are others. The chosen specimens represent widely different characteristics. Variations in magnetic properties as exhibited by crystallised, compact or massive specimens and detached particles, and also the effects of heating have been studied.

* "Wied. Ann." Vol. LII., pp. 749-757, 1894.

† "L'Ecl. electr." Vol. VII., p. 487, 1896. Vol. VIII., p. 56, 105, 1896. "Jour de Phys." 3rd Series, Vol. V., p. 435, 1896.

‡ "Jern-Kontorets Annaler," New Series, 1903, p. 363.

§ B. A. Report, Edinburgh, 1892. "The Electrician," Vol. XXIX., p. 432, 1892.

|| "An de Chim. et de Phys." Vol. V., Series 7, p. 289, 1895.

¶ Physical Society "Journal," Vol XIX., May, 1904.

2. *Instrumentation.*

The type of instrument which has been used for the work described in the present communication, involving as it does the ring method with ballistic galvanometer, was chosen for the following reasons. When the susceptibility is of the order 0·15 or over, the uncertainty of the correction for " end effect " with short specimens becomes serious if the magnetometer is employed. It was not possible to obtain specimens in all cases of greater length than about 4 cm., and an area of cross-section of about 1 sq. cm. was about the practical minimum. An instrument which has formed the subject of a communication to the Royal Society* in connection with recent work carried out for the Geological Survey and Museum begins to have limitations when used to measure susceptibilities of the order 0·15 or over. The instrument actually used is capable of measuring both susceptibilities of a high order and also those so small that the magnetometer can be brought in with accuracy as a check. It has thus been possible to obtain experimental evidence of the accuracy obtainable.

3. *The Ring Magnet.*

The electromagnet which has been used in the experiments for the measurement of susceptibility consists of a ring built up of stampings of stalloy, having an air-gap 2 cm. wide with its sides parallel to a diameter. The specimen is 4 cm. long and has a square section 1 cm. across each side, and is inserted in rectangular holes cut in the pole-pieces of the electromagnet, thus bridging across the air space. Each of the stampings of stalloy has a thickness of 0·32 cm. and they are nine in number. The internal diameter is 7·62 cm. (3 in.) and the external diameter 12·7 cm. (5 in.) The three central stampings, with two additional thin ones to make up the required thickness, are provided with the square recesses above mentioned, and are permanently fixed to the lower three stampings. The upper three stampings are fixed together and can be lifted so as to allow of the insertion of the test piece and then dropped into position, thus closing in the test piece which is then surrounded by the metal to a depth of 1 cm. at each end. Surrounding the ring is a coil of insulated copper wire which is wound on a circular former sufficiently wide near the air-

* "Phil. Trans." R. S., A. Vol. 219 (Appendix), 1919.

gap, to allow of the tilting of the three upper stampings, and also provided with a gap sufficiently wide to allow of the insertion of the specimen. Counting from innermost to outermost the turns of wire, which is wound in three layers, are 146, 140, 126, making a total of 412. The copper wire has a diameter of 0·813 mm., insulated with a double layer of cotton, and the resistance of the winding is 2·25 ohms.

Calculations have been made to find the relation between the current in amperes (C) in the winding and the magnetising force H, produced in the gap. The total line integral of the magnetising force due to the current C is connected with the magnetic induction when no specimen is in the gap by the following equation :—

$$\frac{4\pi nC}{10}=\frac{l_1 I}{A}+l_2 f\left(\frac{\nu I}{A_2}\right),*$$

where n is the number of turns on the ring=412,
l_1 is the length of the air-space=2 cm.,
A is the area of the air-space,
l_2 is the mean length of the lines of magnetic force in the ring=33·9 cm.
A_2 is the area of cross-section of the ring.
ν is the ratio of the induction in the ring to the induction in the air-space.
f is the function defining the magnetising force in terms of the induction in the case of stalloy.

Assuming that the effective area of the air-space is equal to the area of cross-section of the ring, and that the leakage coefficient $\nu=1$, indicating that there is no leakage of the lines of magnetic force, the above equation becomes

$$\frac{4\pi nC}{10}=l_1 \, . \, B_1+l_2 f(B_2),$$

where B_1 and B_2 are the magnetic inductions in the air-space, and ring respectively. In Table I. the function f which defines the magnetising force H_2 in terms of the magnetic induction B_2 is given in the first two columns.† From the assumptions made it follows that $B_1=B_2=H_1$ the magnetic force in the air-space. From the values of H_2 and the mean length, l_2, the line integral of the magnetising force in the stalloy has been calculated. For the air-space the values of $l_1 H_1=l_1 B_2$ have

* "Phil. Trans." R.S. 1886, p. 331.
† Roy. Soc. "Proc." A. Vol. LXXX., p. 548, 1908.

been found, and by addition the total line integral, which is equal to $\frac{4\pi nC}{10}$ has been obtained. In this manner the relation between the current C and the magnetic force H_1 has been

TABLE I.—*Stalloy Magnet Ring.*

Curve of induction for stalloy.		l_2H_2 l_2=33·9 cm. stalloy.	No specimen in air-gap.			Ratio of force in gap to amperes	Ratio $l_2H_2/$ $l_1H_1+l_2H$
H_2	B_2		l_1H_1 l_1=2 cm	$l_1H_1+l_2H_2$	Amperes C	H_1/C	per cent.
			Air.				
0·000474	0·1267	0·0161	0·253	0·269	0·00052	244	5·99
0·000739	0·1918	0·0251	0·384	0·409	0·00079	243	6·14
0·00267	0·674	0·0905	1·35	1·44	0·00278	235	6·28
0·00357	0·937	0·121	1·87	1·99	0·00385	243	6·08
0·00695	1·870	0·236	3·74	3·98	0·00769	243	5·93
0·01286	3·60	0·436	7·20	7·64	0·0148	243	5·71
0·0251	8·25	0·851	16·5	17·35	0·0335	246	4·91
0·0358	13·02	1·214	26·0	27·21	0·0526	247	4·46
0·080	38·0	2·710	76·0	78·71	0·152	250	3·44
0·157	94·1	5·32	188	193·3	0·374	252	2·76
0·245	171·0	8·31	342	350·3	0·677	253	2·37
0·312	269·0	10·6	538	548·6	1·06	254	1·93
0·420	629·0	14·2	1,260	1,274	2·46	256	1·11
0·506	1,063	17·2	2,130	2,147	4·15	256	0·80
0·677	2,245	23·0	4,490	4,513	8·72	257	0·51
1·354	6,050	45·9	12,100	12,146	23·5	257	0·38
2·130	8,200	72·2	16,400	16,472	31·8	258	0·44
3·26	9,810	110·5	19,600	19,710	38·1	258	0·56
5·71	11,500	194	23,000	23,190	44·8	257	0·836
16·20	13,480	549	27,000	27,550	53·2	253	1·99

$4\pi nC/10 = l_1B_1 + l_2f(B_2)$. Assumes area of air-space and stalloy are equal and tha there is no leakage.

l_1=2 cm. l_2=11·43π−2=35·9−2=33·9 cm. n=412 No. 21 S.W.G. copper wire

calculated, and their ratio should be a constant if the line integral in the stalloy were vanishingly small. The figures show that the ratio is a maximum when the magnetising force is such that the permeability in the stalloy is a maximum, and that it does not seriously deviate from a constant value.

4. *Experimental Details.*

In the earlier stages of the investigation a galvanometer of the needle type was employed, but owing to the disturbances set up by the ring magnet (even when remote from the galvanometer) and to other local causes, an instrument of the moving-coil type was substituted. This galvanometer had a resistance of 25.7 ohms and a periodic time of 5·95 seconds,

and its sensibility was such that a steady deflection of one scale division at a distance of one metre was produced by a current of $3{\cdot}77 \times 10^{-8}$ ampere.

An exploring coil consisting of 120 turns of fine silk-covered copper wire was wound on a square former of thin card, which allowed of the specimen being inserted with considerable closeness of fit. The coil was connected to the galvanometer, and an adjustable resistance was included in the circuit to limit the deflections. Deflections were obtained on reversal of the magnetising current C, firstly with the exploring coil supported on a wooden core having the same dimensions as the actual specimens, and secondly when the wooden core was replaced by the test piece itself. The ratio of the deflections, allowing for the alteration in the resistance when necessary, and correcting for the air space occupied by the card former, has been taken to be the permeability; and hence the susceptibility has been obtained.

The assumption above made as to the ratio of the deflections is justified on the following considerations. Firstly, an exploration of the magnetic field in the gap showed that it was substantially constant over the whole area. It was in fact about 5 per cent. weaker in the region near the edge of the air space than at the centre. On reversing a current of 1·02 ampere in the magnetising coil the value of H, the magnetic force in the air space was found to be 248 C.G.S. units. Thus the ratio of this force to the current is 244, and Table I. indicates a ratio of about 254. It does not appear necessary to give other illustrative cases, and it is sufficient to notice that a fair agreement can be obtained. As a further test the susceptibilities of certain specimens were obtained by the magnetometer, and when placed in the ring magnet substantial agreement was found. For example, a piece of Manchurian magnetite, obtained by permission from specimen 9,500 in the collection of the Geological Survey and Museum, when tested in the magnetometer with a force of 53 C.G.S. units, had a susceptibility of 0·118. When tested in the ring magnet, the susceptibility was found to vary from 0·11 to 0·12 when the magnetic force had about the same value.

This method of inserting the test piece in a magnetic circuit as above described and testing it for susceptibility, obviously depends to some extent for its accuracy upon the closeness of fit with the pole-pieces, and the authors do not wish to claim for the method a greater accuracy in the result than

TABLE II.

$H_{max.}$	Traversella Piedmont, No. 712.	New York.	Hey Tor, Devon.	Penryn, Cornwall, 242A.	Penryn, Cornwall, 242C.	Arkansas, U.S.A., No. 1.	Arkansas, U.S.A., No. 2.	Altenfjord, Norway.	Lake Champlain New York.	South Manchuria, No. 9,500.	Magnetite-calcite, Aran.	Manganese Steel.
1·25	1·68	...	...	...	...	...	...	...	...	...	...	...
3·75	2·06	0·566	...	...	...	...	...	...	...	...	...	...
10·5	2·47	0·892	0·52	0·235	0·1515	0·139	0·138	0·133	0·119	0·085	0·104	0·226
17·0	3·07	1·251	0·668	0·246	0·155	0·179	0·163	0·147	0·147	0·105	0·108	0·272
27·2	3·09	1·44	0·741	0·265	0·162	0·186	0·169	0·171	0·151	0·105	0·117	0·272
56·0	2·74	1·37	0·892	0·279	0·183	0·207	0·179	0·169	0·147	0·107	0·118	0·311
84·0	2·44	1·25	0·805	0·341	0·204	...	0·195	0·217	0·151	0·122	0·1275	0·387
112·0	2·09	1·108	0·738	0·382	0·222	0·236	0·205	0·248	0·148	0·117	0·119	0·428
175·0	1·51	0·89	...	...	...	...	...	...	...	...	...	...
203·0	1·37	0·816	0·55	0·487	0·27	0·313	0·277	0·27	0·131	0·124	0·118	0·527
257·0	1·14	0·702	0·469	0·492	0·293	0·352	0·33	0·259	0·127	0·116	0·109	0·535
388·0	0·89	...	0·354	0·437	0·30	...	0·327	0·218	0·113	0·106	0·099	0·433
525·0	0·60	0·43	0·287	0·371	0·273	0·305	0·292	0·188	0·093	0·097	0·085	0·376
756·0	0·45	...	...	0·298	...	0·268	...	...	...	0·09	...	0·310

TABLE III.

No.	Description.	Sp. gr. of specimen.	H_{max}	Deflection on reversal θ.	Deflection on break θ_1.	$\theta/2-\theta_1$ / $\theta/2$ per cent.	Magnetising force at max. suceptibility.	Max. susceptibility.	Coercive force for $H_{max.}=525$.	$B_{max.}$ for $H=525$.	Ergs per cycle per cb. cm. for $H_{max.}=525$.	Intensity of magnetisation retained after being in a field of 18,000 C.G.S. units. After 3 hours.	After 6 days.
712	Traversella (crystal) ...	5·06	567	260	114	12·3	22·5	3·12	12·2	4,495	23,200	2·81	1·84
...	New York... ...	4·86	569	325	135	16·7	31·5	1·46	16·8	3,270	14,000	7·45	6·75
...	Hey Tor, Devon ...	4·30	525	235	99	15·4	49·0	0·90	23·8	2,528	15,300	7·6	2·3
242A	Penryn, Cornwall...	4·56	525	260	80	38·5	236·0	0·49	110·0	2,680	81,600	60·6	57·8
242C	Penryn, Cornwall...	4·59	525	227	76	32·8	368·0	0·31	95·0	2,300	58,000	48·2	48·0
1	Arkansas	4·68	525	247	74	39·8	298·0	0·363	150·0	2,580	89,600	69·7	66·3
2	Arkansas	4·74	525	238	72	39·5	315·0	0·348	150·0	2,450	81,800	68·7	65·2
...	Altenfjord, Norway	4·02	525	239	94	21·0	175·0	0·272	65·0	1,760	13,900	27·8	24·7
...	Lake Champlain, New York ...	4·14	525	221	102	7·3	43·8	0·172	15·2	1,112	720	3·08	2·34
9,500	S. Manchuria ...	3.40	525	349	160	8·2	140·0	0·127	31·0	1,132	1,660	6·12	5·35
W30	Magnetite-calcite, Aran	3·40	525	288	141	2·1	85·0	0·129	12·1	1,080	151	1·89	1·14
...	Manganese steel ...	7·68	788	221	70	36·4	228·0	0·54	108·0	3,012	68,800	62·0	49·7

that to which it is entitled. It is difficult with rock specimens to obtain intimate contact at all points between them and the metal, as they are easily broken or chipped. The procedure was first to cut the test piece from the hand specimen—and in this connection we wish to express our thanks to Dr. W. G. Gordon, Head of the Geological Department of King's College, London, for allowing us to make use of his rock-cutting apparatus—and then to grind down the specimen with carborundum powder in water until it just entered the gaps in the pole-pieces. In this manner very fair contact was made and the

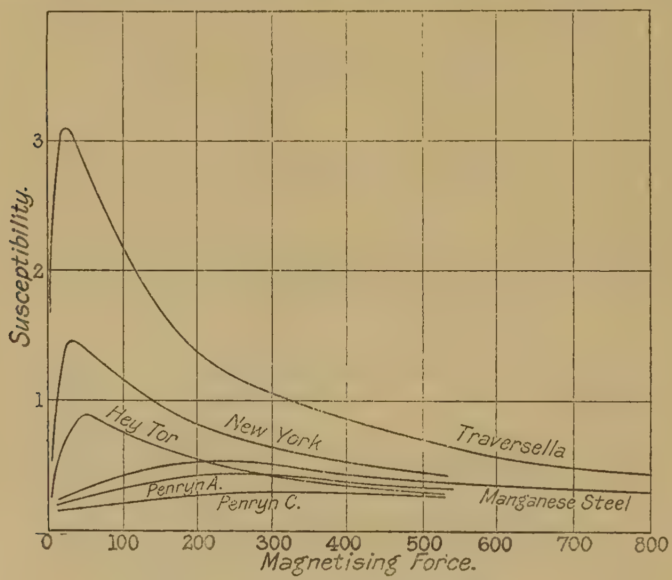


FIG. 1.—K–H CURVES FOR MAGNETITE.

tests show that the results are reliable. Specimens, on being taken out and replaced, when either turned over or turned end to end, gave very closely the same result.

The results obtained are tabulated in Tables II. and III., and illustrated by curves in Figs. 1 and 2. Table II. gives the susceptibility (κ) for all the specimens examined, together with the values of the magnetising force H in C.G.S. units. Table III. gives the specific gravity of the bars actually used in the ring magnet. It also gives information as to the retentivity, maximum susceptibility and the force at which it

occurs, coercive force, dissipation of energy due to the reversal of the magnetising force, and finally the variation of the intensity of magnetisation retained after an application of a force of about 18,000 C.G.S. units obtained by placing the specimen between the coned pole pieces of a powerful electro-magnet. The specimens will now be considered in detail, roughly in the order of their maximum susceptibility.

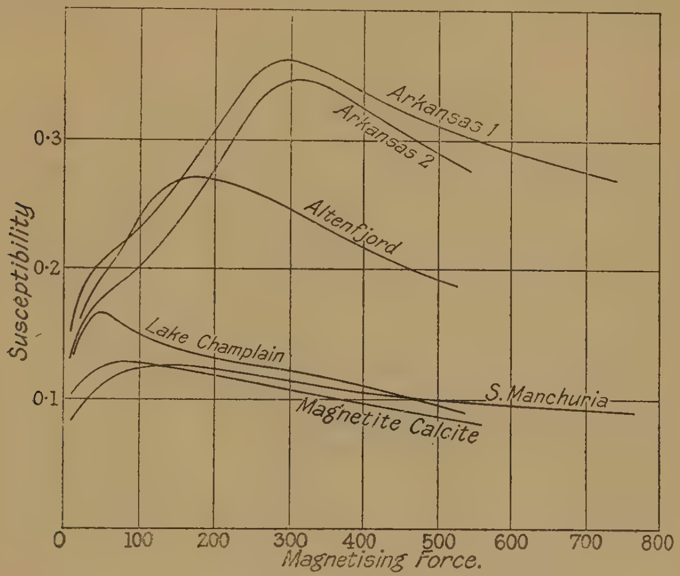


Fig. 2.—K–H Curves for Magnetite.

Traversella, Piedmont. No. 712, *G. S. & M.*

The specimen bar was cut (by permission of the Director of the Geological Survey and Museum) at right angles to the principal axis of a large dodecahedral crystal. This material was extremely brittle with almost vitreous fracture exhibiting a bright semi-metallic lustre. Its specific gravity, 5·06 is the highest of any of the various magnetites examined. It also furnishes the highest figure for susceptibility giving a maximum of 3·12 for a force of $H=22{\cdot}5$. Its coercive force 12·2 is the smallest of any of the specimens of true magnetite and in consequence its retained magnetism is the lowest.

The above value for the susceptibility is considerably below the results obtained by P. Weiss, as exhibited in his curves

of magnetisation. This may possibly be attributed to a difference in the nature of the specimen.

Small fragments after heating in air to about 1,000°C. appeared to undergo no material alteration in magnetic properties, but the authors have not been able to carry out experiments on a sufficiently large test-bar to decide this point fully.

The values obtained using magnetising forces ranging from 1·25 to 756 C.G.S. units will be found in column 2 of Table II., and the curve showing the variation of K with H in Fig. 1.

New York State. (Exact locality not known.).

This is a very uniform material of specific gravity 4·86, very hard and compact with no indication of any cleavage planes. Its fracture is irregular, bright with numerous glistening points and semi-metallic lustre. It is characterised by a high maximum susceptibility 1·46 and small coercive force, coming in these respects second to the Traversella crystal. As will be seen from the table its permanent magnetisation only amounted to $I=7{\cdot}45$, notwithstanding that while in the field of the electromagnet, the value of B in this material is very high.

After having been raised to a temperature of about 1,000°C. and allowed to regain atmospheric temperature, the magnetic susceptibility for a given force ($H=53$) was found to be unchanged; and there was no measurable alteration in its specific gravity.

Hey Tor, Bovey Tracey, Devon.

A specimen belonging to Prof. H. Louis, of Newcastle-on-Tyne, exhibited great irregularity in the distribution of magnetite. The latter consisted of minute crystals mixed with earthy material. Its fracture was dull and earthy with some sparkling points due to the small crystals. The density varied considerably in different parts from 3·89 to 4·30.

Its susceptibility was as high as 0·9 for forces of about 50 units, and its coercive force relatively small (23·8). The magnetite which it contains belongs therefore to the class having high susceptibility and but little retentivity, the maximum value of I being only 7·6 three hours after having been placed in a field of 18,000 units; this fell in four days to only 2·3. Heated to about 1,000°C. in air and allowed to cool, it showed no tendency to disintegrate and only lost 0·38 per cent.

of its weight, which loss may have been water in its pores, and it showed no change in specific gravity. But although there was very little alteration in appearance or weight its susceptibility for a given force was found to have decreased 11·4 per cent., and its residual magnetism to a somewhat greater extent. This furnishes an example of a magnetite that distinctly diminishes in susceptibility and coercive force after being strongly heated and is an interesting contrast to the next variety.

Penryn, Cornwall. No. 242 G. S. & M.

The specific gravity of different pieces from the same specimen varied from 4·50 to 4·65. It is a hard, strong material with a dull fracture and contains small inclusions of a red-brown softer substance, but otherwise compact. This softer substance was found to consist mainly of ferric oxide and ferrous carbonate. When tested in its natural state, its maximum susceptibility had the value of 0·31 for a force of 368 C.G.S. units. After heating to about 1,000°C. the susceptibility rose to 0·492 for $H=235$. In order to exclude any possibility of reduction by furnace gases specimens were heated in an electric furnace in air for 10 minutes to a temperature of 1,000° C. and showed the same marked increase in susceptibility, the maximum of which occurred at a smaller force than for the unheated specimens.

As this kind of magnetite was the only one amongst the many varieties examined by the authors which showed a marked increase in susceptibility, it was thought probable that this might be due to the conversion of some of the ferrous carbonate and ferric oxide existing, as above mentioned, as small inclusions throughout its mass, into magnetic oxide on heating to 1,000°C.

This view was rendered the more probable from consideration of the result of an experiment in which a specimen of this magnetite was heated to about 520°C. for 100 hours to ascertain if prolonged heating to just below the temperature at which magnetite loses its magnetic properties would produce any change. It was found that not only did it fail to show any appreciable increase, but when afterwards it was heated for one hour to 650°C., it showed no further change. Finally, heating it to 1,000°C. in the electric furnace produced no increase in its susceptibility, which after all these various

heatings showed no increase, but had actually diminished about 5 per cent.

As a control two small pieces from the same portion of rock were heated rapidly to 1,000°C. and kept at that temperature for five minutes and allowed to cool in air ; they both showed a marked increase in susceptibility of over 60 per cent. The density was slightly raised by this heating process from 4·57 to 4·60, and in a previous case from 4·55 before heating to 4·63 after heating. In these two cases the percentage loss in weight was 1·14 per cent. and 1·07 per cent. respectively. The fact that prolonged heating in air to a temperature of 520°C. prevented the increase in susceptibility may probably be due to the oxidation and decomposition of the ferrous carbonate yielding ferric oxide, while when rapidly raised to 1,000°C. it was transformed (in conjunction with some of the ferric oxide mixed with it) into magnetic oxide (Fe_3O_4). Both the loss of weight and increase of density as well as the great increase in susceptibility receive explanation by this theory, with which the changes of colour and appearance also agree.

G. Folgheraiter* examined the effect of heating on various volcanic rocks and found in general an increase in magnetic properties especially in the amount of retained magnetism, and considered this to be due partly to the conversion of non-magnetic into magnetic substances, and partly to the orientation of magnetite crystals. Brunhes and David† have noticed that the flow of hot lava over clay transformed it into a brick-like material that is magnetic.

With a view to testing the effect of heat upon known material a mixture of kaolin with an equal weight of finely powdered native ferrous carbonate (Chalybite) was made into a stiff clay with water, moulded into bars, dried and finally heated gradually to a temperature of about 900°C.

A rectangular bar 4 cm. × 1 cm. × 1 cm. after being baked was used in the ring magnet and gave a value for K of 0·0224 in a field of 284 units, and 0·021 for $H=178$. Before heating the value of K was too small to measure by this method, but by the use of the magnetic balance it was found to have a susceptibility of 0·000218. Hence its susceptibility was increased about 100-fold by heating to a bright red heat.

In one experiment with a cut bar of this Penryn magnetite the specimen was heated to 1,000°C. and allowed to cool in a

* "Rend. Acc. dei Lincei," Jan. 4 and Feb. 4, 1895.

† "Comptes Rend." 137, p. 975, 1903.

vertical position standing on a piece of asbestos cardboard. When cold its lower end was found to be, as was to be expected, a N-seeking pole, and the intensity of magnetisation it had acquired under the influence of the vertical component of the earth's field was found to be 1·48 C.G.S. units. Experiments conducted on the same bar before heating showed that it required a force of 95 units to impart nearly the same (1·40) intensity of permanent magnetism. Evidently during the cooling process the susceptibility reaches a high value and the magnetism then induced is retained permanently after cooling.

This fact may have an important bearing on the question of the intensity of magnetisation of beds of magnetite in a natural state.

The values for the susceptibility with different magnetising forces for this kind of magnetite are given in columns 5 and 6 of Table II., A being the substance after having been heated to 1,000°C. and C being obtained from a bar in its natural state, unheated.

Arkansas (*I. & II.*).

Two specimens obtained from independent sources (one of the specimens was kindly given to the authors by Dr. Gordon) showed a marked similarity in properties. It consists of a hard compact material with irregular cleavage planes and almost vitreous fracture in some directions while exhibiting a silky lustre in others.

The specimen bar called "Arkansas I." had a specific gravity of 4·68 and "Arkansas II." varied from 4·70 to 4·82, the actual bar experimented with having a specific gravity of 4·74. Both samples showed distinct permanent magnetisation when received and portions of a specimen exhibited a high intensity of magnetisation (I.) of the order of 30 C.G.S. units. As will be seen in Table III. the highest sub-permanent magnetisation exhibited by one of the 4-cm. bars amounted to 69·7, but this was after exposure to a field of about 18,000 units. After being magnetised in a field of 806 the intensity of retained magnetism was 44·5 immediately after removal, and fell to 43·1 in 12 hours. It is evident that a magnetisation of even 30 C.G.S. units for I can only be caused either by a field strength enormously exceeding the earth's force, or by intense local forces such as lightning flashes, as suggested by Pockels* in connection with the magnetisation exhibited by

* "Annal. Phys. Chem.," 63, pp. 195-201, 1897.

specimens of basalt, or by molecular changes accompanying heating and cooling under pressures with which we are at present unacquainted.

The coercive force exhibited by both specimens is the same, viz., 150 units for H max.$=525$, and is higher than for any other magnetite the authors have examined.

A specimen heated to 1,000°C. showed a minute increase in weight after regaining room temperature, but only to the extent of 0·08 per cent.; no change in the density was detected. The susceptibility, which for the specimen was 0·202 for a field of 52 C.G.S. units before heating, was found after heating to have increased for the same force to 0·286, being an increase of 41·6 per cent. Its residual magnetism was also greater, but its coercive force was very materially reduced, so that the retained magnetism was less than half that exhibited by other portions of the same unheated specimen after being in an intense field. While the after effects of heating in this case are to increase K and diminish coercive force (as with hard steel), in the case of Penryn magnetite "242" both K and coercive force were permanently increased.

The dissipation of energy in ergs per cycle per cubic centimetre reaches the high figure of 89,600 for this material for $H_{max}=525$ units, and is the highest recorded in Table III. column 12.

Altenfjord, Norway.

A specimen of magnetite from the above locality was kindly placed at the disposal of the authors by Prof. H. Louis. It was found to be essentially an aggregate of magnetite and felspar having an irregular but crystalline fracture. Its density varied from 4·0 to 4·2 following the variation in proportion of magnetite to felspar.

Its susceptibility for different magnetising forces will be found in column 9 of Table II. Its maximum susceptibility (Table III.) reached the comparatively low value of 0·272 for a force of 175 C.G.S. units.

Heating for 10 minutes to a temperature of about 1,000°C. had no subsequent effect on its weight or specific gravity; the susceptibility of a specimen after this treatment was found to be slightly increased, its value for a given force rising from 0·227 to 0·239, or about 5·3 per cent. increase. The retained magnetisation, however, was found to be substantially the same before and after heating, so that there was no appreciable change in its retentivity.

Lake Champlain, New York State.

One of the varieties of magnetite from this locality was supplied to the authors by Prof. H. Louis and was found to be an aggregate of magnetite and iron pyrites; it had an irregular glistening fracture in which the pyrites could be easily detected by its lighter colour. Its density was found to vary very considerably, 4·10 to 4·62 being the limits for different parts of the same hand specimen, the denser parts being visibly richer in magnetite.

Its magnetic values are recorded in Tables II. and III., and its curve $(K-H)$ in Fig. 2. Its low maximum in susceptibility 0·172 is no doubt due to the variable admixture of pyrites.

The effect of heating this substance to 1,000°C. is to disintegrate it to a greater or less extent, owing to the decomposition of the pyrites. There is a slight loss in weight and decrease in density, the actual figures for the latter in one case being 4·62 before heating and 4·60 after. The susceptibility for the same force was found in a specimen which did not disintegrate to any extent to be diminished about 7·5 per cent., the retained magnetisation being similarly reduced.

South Manchuria. *No.* 9500 *G. S. & M.*

A specimen of magnetite from the above locality was obtained by permission of the Geological Survey and Museum from their collection. It is essentially a schist having a slaty cleavage in which the laminated structure is easily detected. Owing to its comparative poorness in magnetite its specific gravity is the lowest yet dealt with, being only 3·40, and remarkably constant in different parts of the same specimen. For the same reason its susceptibility is the lowest the authors have found for any rock which can be described as "Magnetite." Its susceptibility, which is 0·085 for $H=10{\cdot}5$, passes through a maximum of 0·127 for $H=140$, falling again to 0·09 for $H=756$. As the $K-H$ curve in Fig. 2 shows, its susceptibility is more nearly a constant than any other variety examined. In this connection the Authors have not yet found in any specimen of magnetite so low a value of the susceptibility (0·016) as that given by Allan.* This, however, may be due to the low value of the magnetising force which he employed.

Heating to 1,000°C. in an electric furnace caused a small loss of weight amounting to 0·07 per cent., probably due to

* Loc. cit.

loss of water; the specific gravity was subsequently found to be unchanged and its magnetic properties were not materially affected. There was a very slight increase in susceptibility, but so small that it falls almost within the limits of experimental error.

Its magnetic values are recorded in column 11 of Table II. and also in Table III.

Magnetite Calcite. (*W.* 30) *Aran, Wales.*

A small specimen of the above rock having been supplied to the authors by the Geological Survey and Museum, it has been included in this series, although strictly speaking it can hardly be designated as a Magnetite. It consists of a matrix of calcite in which are disseminated irregularly numerous very small crystals or crystalline grains of magnetite. In spite of the small proportion of magnetite which it contains (a proportion so variable in different parts that an analysis would give only a vague indication) its susceptibility reaches a slightly higher figure than the Manchurian magnetite last mentioned. The small residual and retained magnetisations, the small coercive force—substantially the same as that of the Traversella crystal—all point to the conclusion that what magnetite is present is in the form of the pure crystal.

From the nature of the matrix—calcium carbonate—it is impossible to investigate the effect of very high temperatures on this material.

A 13 *per cent. Alloy of Manganese and Iron.*

A rectangular specimen of this alloy was cut from a ring which was given to one of the authors some years ago by Sir R. A. Hadfield, F.R.S. It was maintained at a temperature of about 530°C. for 100 hours, with a consequent increase in the susceptibility on return to atmospheric temperature. The results of the experiment with this specimen are interesting as they exhibit a similarity to those of some of the specimens of magnetite. In particular, it is interesting to note the relatively high value of the coercive force and considerable retained magnetisation in this annealed manganese steel, as it is commonly asserted without reference to its condition that manganese steel is almost devoid of these properties.

SUMMARY.

The magnetic properties of certain varieties of magnetite as exhibited by crystallised, compact or massive specimens and detached particles have been examined. In each case the susceptibility has been found to vary with the magnitude of the magnetising force after the manner of iron, the relative variation being much more pronounced in the case of those specimens having the higher susceptibility. The maximum susceptibility in the specimens examined occurs at a force ranging from 13 C.G.S. units in the crystal to 368, its magnitude varying from 3·12 to 0·127 C.G.S. units.

The effect of heating has been greatly to increase susceptibility in some cases and in others a negative effect has been produced. In the case of a specimen of Penryn magnetite, the large increase in the susceptibility was traced to the conversion of ferrous-carbonate and ferric-oxide into magnetite.

As bearing upon the intensity of magnetisation of magnetite in a natural state, it may be mentioned this specimen of Cornish magnetite acquired an intensity of 1.48 C.G.S. units on cooling in the earth's field from 1,000°C., whereas in the cold before heating such an intensity required a field of 95 C.G.S. units to impart the same degree of retained magnetisation.

Very high susceptibility in magnetite is never associated with high coercive force or retained magnetisation, the greatest values for the latter exhibited by specimens having an intermediate value of susceptibility of the order of 0·3 or 0·4. Lower susceptibility may be associated with high coercive force, but naturally the retained magnetisation is not very great, owing to the lower maximum of induced magnetisation.

DISCUSSION.

Prof. Louis said that he was not competent to discuss the physical problems involved, but was intensely interested in the practical applications of the work done by the authors of the Paper. He congratulated them on having taken a step towards the solution of the very puzzling questions why magnetite differed so widely from all other minerals in its magnetic properties, and why there were such wide differences in these properties between different specimens of magnetite. The most magnetic magnetite that he had ever met with was in one of the higher peaks of the great mass of magnetite that formed the huge hill of Kirunavaara, where the mineral was so strongly magnetic that on breaking it with a hammer the small chips remained adhering to the face of the mineral by magnetic attraction. The suggestion of the authors that such effects might be due to lightning seemed to fit this case very well. He was not at all surprised at the results obtained from the Penryn mineral; this evidently contained spathic ore, and if this latter were heated even to 500°C. without access of air, the carbonate of iron would be converted into a substance approximating to Fe_3O_4 in composition, and this "artificial magnetite" was strongly magnetic; the presence of Fe_2O_3 was not at all necessary for this action to take place. Carbonate of iron is so easily converted into "artificial magnetite" that this method is quite often used on the large scale for separating spathic ore from such minerals as blende; he (Prof. Louis) had found that in this way it was possible to obtain a certain amount of concentration even in the case of such unpromising material as Cleveland ironstone. He assumed that the Lake Champlain specimen must have been heated with free access of air, so that the pyrites present had been completely oxidised, because pyrites

heated in the absence of an oxidising agent became converted into a sulphide approximating to pyrrhotite in composition, which was fairly magnetic. He hoped that the authors would investigate this particular point and determine under what conditions of heating pyrites gave the most strongly magnetic product. The change began at a comparatively low temperature, and the speaker had met with difficulties in the satisfactory concentration of certain iron ores owing to this effect. The high magnetic susceptibility of the specimens from Traversella was especially interesting, because it was to the ore from this locality—a mixture of magnetite and copper pyrites—that electro-magnetic separation on a commercial scale was first applied. It is quite possible that the high susceptibility was due to these specimens having been cut from an idiomorphic crystal. He sincerely hoped that the authors would continue their extremely interesting researches upon the magnetic properties of minerals.

Prof. A. H. Cox communicated the following: I regret that I am unable to be present at the reading of Prof. Ernest Wilson's Paper, but, although I am not qualified to speak upon the purely physical aspect of Prof. Wilson's researches, I should like to be allowed to congratulate him upon the success of his work upon the measurements of small susceptibilities. The designing of a portable instrument that will measure such small susceptibilities marks an advance which will greatly facilitate work on certain problems, the solution of which appears likely to lead to results of great economic importance. The results so far obtained from the investigation of local magnetic disturbances in certain selected areas, have shed a new light upon the underground structure in one of the most important of our concealed coalfields, and encourage the hope that we have in our hands an additional method of attacking the problem of underground geology. The determinations made by Prof. Wilson gave the necessary clue as to which rocks were responsible for causing the magnetic disturbances in those districts. One of the difficulties encountered during the investigation into the origin of the disturbances was the fact that no susceptibility determinations of such weakly magnetic materials as the rocks involved could be carried out during the course of the outdoor work. Before specimens could be tested magnetically a certain amount of preparation was necessary, and they had all to be sent to Prof. Wilson's laboratory. Now, as the result of his researches, it should be possible to carry out susceptibility determinations at the points where the rocks actually occur, thus effecting a great saving of time, and removing one of the great difficulties in further investigations into the relationship between magnetic disturbance and geological structure. If, as seems probable, such investigations lead to results of national importance, the value of Prof. Wilson's work becomes more than of purely scientific import.

Dr. D. Owen said that the authors' experiments gave interesting information as to the nature of the differences of magnetic quality usually found between various specimens of lodestone. The experimental method, though not quite free from sources of error, appeared to allow of a degree of accuracy satisfactory for the purpose; it would, however, be of interest to have the figure of accuracy stated. In interpreting Figs. 1 and 2 it should be noted that the values of the magnetising force must not be taken too literally. Since many of the specimens were composed of small crystals of magnetite, it is evident that demagnetising forces of uncertain extent must have been present. The fact that the specimen of Traversella showed a much lower value of susceptibility than that obtained by P. Weiss is not surprising, in view of the fact that even good samples of the mineral contain a large amount of impurity, of the order of some 10 per cent. The somewhat low density (5·06) of the specimen supports this view. The explanation proposed by the authors to account for the rise of susceptibility of some specimens after subjection to a rise of temperature appeared reasonable. The ferrosoferric oxide thereby produced is, like magnetite, distinctly ferro-magnetic.

Comparative figures of susceptibility of the artificial and crystalline varieties would be of interest in this connection. In regard to the problem of the high intensity of magnetisation possessed by some specimens of magnetite, have the authors any evidence as to whether a crystal heated above the critical temperature and allowed to cool in a zero magnetic field would show any sign of magnetisation ?

Mr. C. R. DARLING said that magnetic separators were used for other minerals than magnetite, and it was of interest in connection with the effect of heating obtained by the authors, that in some of these cases heating rapidly to 1,000°C. improved the ease with which separation was accomplished.

Mr. POWER said he had been engaged on the magnetic separation of tin and wolfram ores, and had experienced trouble on account of the magnetic product containing about 33 per cent. of tin. On separating this the iron content was found to be very low. Heating to a high temperature did not alter the magnetic properties. Various pickling processes were tried, but, although a little of the iron was got out, there was still enough left to render the tin magnetic. With regard to the pyrites in the Champlain magnetite, if pyrites is heated slowly it is not nearly so magnetic on cooling as when heated rapidly.

Prof. FORTESCUE asked what steel was used in the experiment shown on the table.

Mr. F. E. SMITH said the authors in their formula took the air-gap as being the space between the jaws of the magnet poles. What effect had the recesses in the polé-pieces on the results ? With regard to the magnetic properties of manganese steel, it was well known that this steel had a permeability of approximately unity in weak fields. Employed on the bridge of a ship, for instance, it would be without effect on the compass. If, however, it was subjected to a strong field it became quite magnetic.

Mr. J. GUILD asked if it would not be possible to make observations of the permeability of the specimens at other than air temperatures. The temperature permeability curves would probably show at a glance the answers to many of the points raised in connection with heat treatment, in addition probably to revealing other points of importance.

Prof. E. F. HERROUN, replying to some of the chemical questions that had been raised in the discussion, said that he agreed with Prof. H. Louis that if the Penryn magnetite, or the mixture of kaolin and ferrous carbonate, had been heated out of contact with air, a still greater increase in susceptibility would have resulted ; but that, as all the other specimens had been heated in air, it was a fair and uniform treatment. He also agreed that by heating the magnetite containing pyrites in air some of the latter would be oxidised instead of merely losing some of its combined sulphur. In reply to another question, he said that ferrous or ferric oxides resulting from the decomposition of iron compounds, although paramagnetic, were extremely feeble in comparison with the ferro-magnetic *magnetite*, Fe_3O_4 : crystals of the latter may be formed artificially by using a fusible matrix. Being asked by the President for his distinction between para and ferro magnetic bodies, he said he regarded a ferro-magnetic substance as one which showed a definite K-H curve of susceptibility, while the latter was practically a straight line with paramagnetic bodies for all fields, and its value very small.

Prof. E. WILSON said that Prof. Louis had mentioned how highly magnetic was the magnetite on some of the summits of the well-known Swedish deposit, which had been estimated to contain about 200 million tons. If this magnetisation was due to lightning flashes, it would be interesting to know if the magnetite below the surface was definitely magnetic. In an experiment with bars of Arkansas magnetite and glass-hard steel of the same size, the authors showed at the meeting that, although the bar of magnetite had evidently the greater moment (by its action on a compass needle) its portative force was smaller. In answer to a question by one of

the speakers, he stated that the steel used was a carbon tool steel. He explained that the portative force depends upon the square of the magnetic induction, and that the lower permeability of the magnetite counteracted the higher moment to the extent of making its portative force smaller. Mr. F. E. Smith had remarked upon the value of the permeability of manganese steel being unity, or very nearly so, in weak fields, and that the effect of the earth's field on this steel would not appreciably affect a compass needle. Prof. Wilson said that the value of the permeability of 13 per cent. manganese steel usually quoted was 1·27, and that this figure was obtained by the late Dr. John Hopkinson in the case of a ring. If the surface of the ring is ground away, he had found the permeability of the interior portion had a value of 1·005 in a field of about 100 C.G.S. units. After prolonged heating to 530°C., its permeability may be raised to 8 or 10, or even higher. In regard to Mr. Smith's remark on the influence of the recess in the pole-faces, it is stated in the Paper that the force varies 5 per cent. between the centre of the gap and the edge of the pole-face. If the instrument had to be used for the measurement of very high permeabilities, the closeness of fit would be serious, as is well known in connection with permeameters; but the permeabilities met with in magnetite are not of such a high order as to render the method of test invalid.

XXIII. *The Current-voltage Characteristics of High-voltage Thermionic Rectifiers.* *By* Prof. C. L. FORTESCUE.

RECEIVED MAY 30, 1919.

1. *Introductory.*

THE following notes partake more of the nature of a set of designer's curves than of a record of new work, the conditions prevailing in thermionic rectifiers having been fully described already by Dushman.*

The combination of a high-voltage thermionic rectifier, an alternating-current transformer and a smoothing condenser seems likely to have a wide application as a source of high-voltage direct current. Such an arrangement is well adpated, for example, for X-ray work, for wireless telegraphy and for any laboratory purpose requiring an approximately steady high-voltage unidirectional supply.

The application of these rectifiers for practical wireless work led the author, in collaboration with Mr. C. M. Sleeman, of

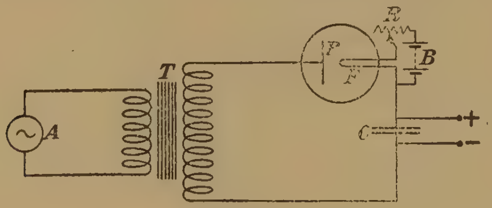


FIG. 1.

Queen's College, Cambridge, to work out the numerical relationships which form the subject matter of this Paper; and, in view of the probability of there being many other applications, it has seemed desirable that the results should be published. These results were required for the development of Wireless Telegraphy in the Naval Service, and this Paper is published by special permission of the Admiralty.

2. *General Arrangement of the Circuits.*

The simplest form of circuit is that shown in Fig. 1. In this figure, A is an alternator and T is the alternating-current transformer necessary to give the high-voltage alternating supply. The thermionic rectifier V consists of the usual

* "General Electric Review," March, 1915.

"plate" or positive electrode, P, and tungsten wire filament, F, for the negative electrode. A high vacuum rectifier is essential for high-voltage working, and the electrodes must be so treated during manufacture that no gas is evolved when the rectifier is in use. The filament is heated by current from a battery, B, which must be highly insulated, or, alternatively, from a separate small alternating-current transformer having a low voltage, but highly-insulated secondary winding. The current in the filament is controlled by the resistance R.

When the filament is at a suitable temperature the electron emission allows of a current flowing during the part of the half cycle when the plate is positive to the filament; but during the rest of the cycle, when the plate is negative to the filament, no current flows so long as the vacuum is sufficiently good. Thus the condenser C becomes charged, and an approximately steady direct current can be taken from the terminals marked + and −. The voltage across the condenser C

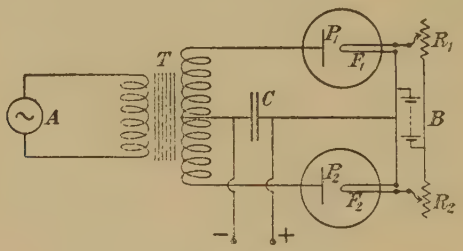


FIG. 2.

will depend upon the secondary voltage of the transformer, and upon the ratio of the direct current supplied to the saturation electron current obtainable from the filament of the rectifier.

An improved arrangement is that shown in Fig. 2, where two rectifiers are used. The operation of this combination is exactly the same as in the previous case, except that the rectifiers come into action alternately, one during one half cycle and one during the other. This arrangement has been described as the "Bi-phase" system of connecting up the rectifiers.

As compared with Fig. 1, the transformer secondary voltage must be twice as great, and the electron emission from each filament one-half, for the same approximately steady voltage and output.

Many other similar circuits can be used. For example, where three-phase power is available, three or six-phase connections, with three or six rectifiers, may be used, with the corresponding reduction of the size of smoothing condenser for a given uniformity of the direct-current supply.

With arrangements of this kind there is no difficulty in obtaining a supply of steady direct current up to values of an ampere, or more, at voltages up to 10,000 volts.

3. *A More Detailed Consideration of the Action of the Circuits.*

Consider the steady conditions in which a steady direct current of I_0 amperes is being supplied at a P.D. of V_0 volts, both current and voltage being approximately uniform.

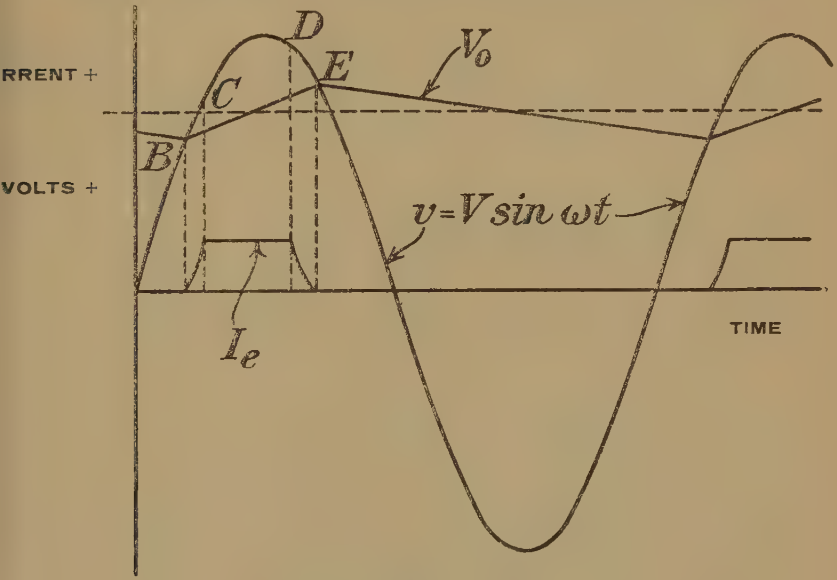


Fig. 3.

Let $v = V \sin \varpi t$ be the transformer secondary voltage, assumed to be approximately sinusoidal, and let I_e be the saturation electron current. The action of the circuit for one cycle is shown diagrammatically in Fig. 3. Until the transformer voltage rises above the condenser voltage V_0, as at B in Fig. 3, no current passes. From B to C the effective voltage across the rectifier is gradually increasing, with a corresponding increase of the current flowing. At every instant the current

varies as the 3/2th power of the excess positive voltage of the plate. At some point, C, however, the saturation value I_e will be reached. From that point the current remains constant until the point D is reached, there being a corresponding small rise of the voltage V_0 during this period. But from the instant D onwards the voltage between plate and filament is falling below that required to produce the saturation current, and consequently from that point onwards the current falls until the instant E is reached, at which there is no voltage between plate and filament, and the current ceases. For the remainder of the cycle there is no conduction through the rectifier, and the voltage at the condenser gradually falls to the value it had at the instant B; after which the process is repeated for the next cycle.

For steady conditions the quantity of electricity passing through the rectifier during the conductive part of the cycle must be equal to the quantity flowing out from the condenser C as a steady current during the whole cycle. Thus, for a given steady supply voltage, V_0, the greater the steady current required the greater must be the saturation current I_e; or, alternatively, for a given value of I_e and of the alternating-current supply, the greater I_0 is, the lower will be the voltage V_0, since the electron current must flow for a longer time to supply the larger quantity per cycle. The maximum value of I_0 is $\frac{1}{2}I_e$, when the voltage V_0 will be zero.

4. *Approximate Calculations.*

The following approximate method has been found to give good results, especially where the values of V and V_0 are both large compared with the voltage V', required to drag away the saturation electron current. In a later paragraph corrections are given which can be applied to the formulæ when a higher degree of accuracy is required.

It is assumed that the periods BC and DE of Fig. 3 are negligibly small, so that the electron current can be supposed to rise to its saturation value at the instant at which the transformer voltage reaches the average steady voltage V_0; and to fall to zero again at the instant when the transformer voltage is again equal to the condenser voltage. Under these assumptions the curve of the current through the rectifier becomes a rectangular one of constant maximum value I_e.

(*a*) *The Capacity Required in the Smoothing Condenser.*—Let the value of the angle ϖt corresponding to the instant at which

$v=V_0$ be θ—*i.e.*, $\theta=\sin^{-1}\frac{V_0}{V}$. Then, if T is the time for one complete cycle of the alternating supply, the full electron current is flowing for a period of $\frac{\pi-2\theta}{2\pi}.T$ seconds per cycle. The quantity of electricity passing into the condenser during the conductive period is therefore $I_e.\frac{\pi-2\theta}{2\pi}T$. The quantity leaving during this period is $I_0.\frac{\pi-2}{2\pi}T$. The quantity to be stored in the condenser for use during the non-conducting period is therefore $(I_e-I_0)\frac{\pi-2\theta}{2\pi}T$.

But for steady conditions, since the quantity per cycle flowing into and out of the condenser must be the same

$$I_e.\frac{\pi-2\theta}{2\pi}T=I_0T, \text{ or } I_e/I_0=2\pi/(\pi-2\theta).$$

The maximum change of charge in the condenser during the cycle is, therefore, $I_\theta.\frac{\pi+2\theta}{2\pi}T$.

If the permissible variation of the condenser is aV_0, where a is a fraction having values of from about 0·1 downwards, then, if C is the condenser capacity,

$$C=\frac{I_0}{V_0}.\frac{\pi+2\theta}{2\pi}.\frac{T}{a}=\frac{I_0}{V_0}.\frac{1}{af}.\frac{\pi+2\theta}{2\pi},$$

where f is the frequency of the alternating-current supply.

For example, if $I_0=0·05$ amperes, $V_0=10,000$, $f=50$, $\theta=1$ radian, and $a=0·05$; then $C=1·64$ microfarads.

With the bi-phase arrangement, the capacity required is found by a similar argument to be

$$C=\frac{I_0}{V_0}.\frac{1}{af}.\frac{\theta}{\pi}.$$

With the numerical values as just previously given, the capacity of the condenser to give the same smoothing effect would be 0·64 microfarads, showing the advantage of the bi-phase system. With the multi-phase arrangements, this advantage is still further accentuated.

The overall dimensions of the condenser are approximately proportional to CV_0^2 which is equal to $\frac{I_0 V_0}{af} \cdot \frac{\pi+2\theta}{2\pi}$ or $\frac{I_0 V_0}{af} \cdot \frac{\theta}{\pi}$ for the single-phase or bi-phase arrangements respectively. Hence for a given frequency of the alternating-current supply the dimensions of the condenser required for a given smoothing effect are dependent only on the output power. Thus with high power it becomes almost necessary to use either high frequencies, *i.e.*, frequencies above the ordinary commercial frequencies, or to use the multiple-phase systems. As will be pointed out later, there is no loss of regulation or of overall efficiency involved in using the multiple-phase system. It is slightly more complicated in that there are several filaments to be adjusted instead of only one; and the initial expense will be slightly greater, since it costs somewhat more to make two small rectifiers than one large one of the same output. When nearing the limit of the power that can be dealt with in a single rectifier, this latter objection does not hold, however, as the cost of the large rectifiers goes up very rapidly when nearing the limit of size.

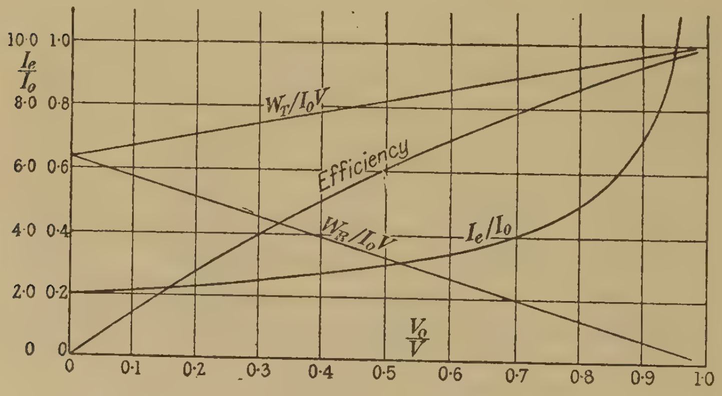


FIG. 4.

(*b*) *The Ratio of the Output and Saturation Currents.*—It has been shown just previously, from consideration of the fact that the quantity of electricity flowing through the rectifier per cycle must be the same as the quantity flowing out of the condenser, that $I_e/I_0=2\pi/(\pi-2\theta)$, where $\theta=\sin^{-1}V_0/V$. Hence

for any assigned value of V_0/V and of I_0, the necessary saturation current I_e can be estimated. A curve of the ratio I_e/I_0 in terms of V_0/V is useful, as given in Fig. 4.

(c) *The Power taken from the Transformer.*—On the same assumptions, the power taken from the transformer W_T is

$$W_T=\frac{\varpi}{2\pi}\int_{\varpi t_1=\theta}^{\varpi t_2=\pi-\theta} I_e \,.\, V\sin\varpi t dt=\frac{I_e \,.\, V\cos\theta}{\pi}=I_0 V\frac{2\cos\theta}{\pi-2\theta}=k I_0 V,$$

where $$k=\frac{2\cos\theta}{\pi-2\theta}.$$

(d) *The Power Expended in the Rectifier.*—The mean power expended in the rectifier is

$$W_R=\frac{\varpi}{2\pi}\int_{\varpi t_1=\theta}^{\varpi t_2=\pi-\theta} I_e(V\sin\theta-V_0)dt=I_0 V\left(k-\frac{V_0}{V}\right).$$

(e) *The Rectifying Efficiency.*—Neglecting the power expended in heating the filament, the efficiency of the rectification is $\dfrac{W_T-W_R}{W_T}=\dfrac{1}{k}\,.\,\dfrac{V_0}{V}$.

Curves of k, $k-V_0/V$ and V_0/kV can be plotted as shown in Fig. 4. These curves show clearly the compromise that has to be made in practice between the ratio V_0/V and the efficiency on the one hand, and the saturation current on the other.

The curve of $k-V_0/V$ is important as it gives the power that has to be dissipated at the plate of the rectifier for any value of I_0V_0. The power dissipation is one of the fundamentals upon which the design of the rectifier depends.

(f) *The Root-mean-square Current taken from the Transformer.*—The saturation current I_e is flowing in the secondary winding of the transformer for a period $\dfrac{\pi-2\theta}{\varpi}$ seconds per cycle. Then if J is the root-mean-square value of this current

$$J^2\,.\,\frac{2\pi}{\varpi}=I_e^2\,\frac{\pi-2\theta}{\varpi},\ \text{or}\ J=I_e\left(\frac{\pi-2\theta}{2\pi}\right)^{\frac{1}{2}}.$$

It is convenient to plot the ratio J/I_0 for various values of V_0/V as is done in Fig. 5.

5. *The Application of these Results to the Bi-phase and Multi-phase Methods of Connecting up the Rectifiers.*

The action of the bi-phase circuit is shown diagrammatically in Fig. 6. The curves of Fig. 4 can be applied directly to this case if the transformer secondary voltage $v = V \sin \varpi t$ is taken as the transformer voltage between outers and the centre point, instead of the voltage across the whole secondary

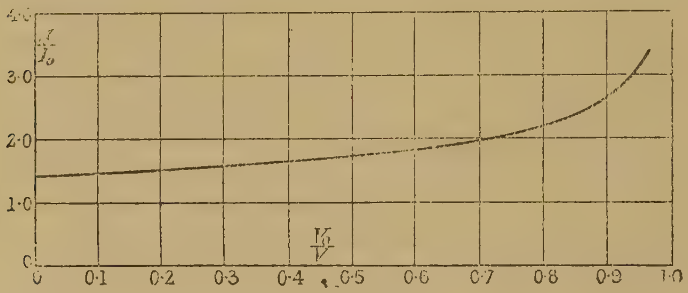


FIG. 5.

winding; and if the ratio I_e/I_0 is halved, I_e being the saturation current per valve, *not* the sum of the saturation currents for the two valves.

For a three-phase circuit the transformer voltage must be taken as between outers and centre point and the ratio I_e/I_0 must be divided by three.

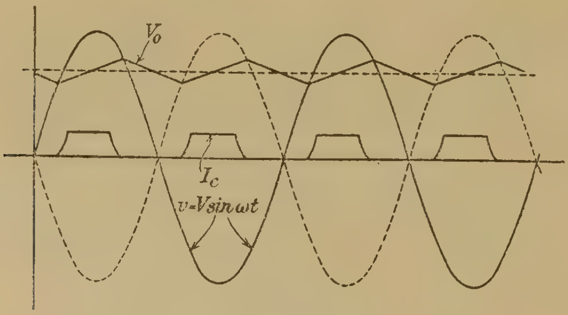


FIG. 6.

In general, for n phases each of voltage $v = V \sin \varpi t$, the curves of Fig. 4 are applicable if the ratio I_e/I_0 is divided by n.

The efficiency will clearly be unaffected by the number of phases because the efficiency curves of Fig. 4 apply to each

individual valve, and therefore to any number of valves, each of which is independent of the rest.

6. *The Over-all Efficiency of the Rectifier.*

From the curves of Fig. 4 it is seen that the rectifier efficiency increases as the ratio I_e/I_0 increases. In practice there is a limit to this ratio arising from the large current required for the heating of the filament and from the large amount of power consumed in the filament. Since the emission per unit area of the filament decreases with decrease of filament temperature, a larger area must necessarily be used with the lower temperature. This means that either the filament must be longer and work at a higher voltage, or it must be of larger diameter and take a larger current.

The characteristics of tungsten filaments have been given by Langmuir.* These characteristics show that the filament watts per ampere of electron emission fall rapidly as the temperature increases. But the life of the filament also falls rapidly as the temperature increases. Consequently, the filament watts per ampere of electron emission is an indication of the life to be expected from the filament of the rectifier.

The overall efficiency of the rectifier, including the filament watts, can be estimated for a series of values of the filament watts per ampere, and overall efficiency curves plotted.

Using the same approximations as before, the output of the transformer is $W_T=kI_0V$. The watts expended in the filament may be expressed as $W_F=wI_e$, where w denotes the watts per ampere of electron emission.

Thus the total input to the rectifier is

$$W=W_T+W_F=kI_0V+wI_e=I_0V\left(k+\frac{w}{V}\ \frac{2\pi}{\pi-2\theta}\right).$$

The output from the rectifier is I_0V_0 and the overall efficiency is, therefore :—

$$\frac{V_0/V}{k+\dfrac{w}{V}\cdot\dfrac{2\pi}{\pi-2\theta}}.$$

The curves of Fig. 7 give the overall efficiency in terms of V_0/V for various values of w/V. The falling-off of the maximum efficiency as the ratio w/V increases, is very marked. With a filament temperature corresponding to 100 watts per

* "Physical Review," N.S., Vol. VII., No. 3, March, 1916.

ampere of electron emission, and a transformer peak voltage of 10,000, the maximum efficiency is in the neighbourhood of 90 per cent. Under these conditions an average life of from 1,000 to 2,000 hours would be expected. But if $w=200$ and $V=4,000$, the maximum efficiency is only about 65 per cent.

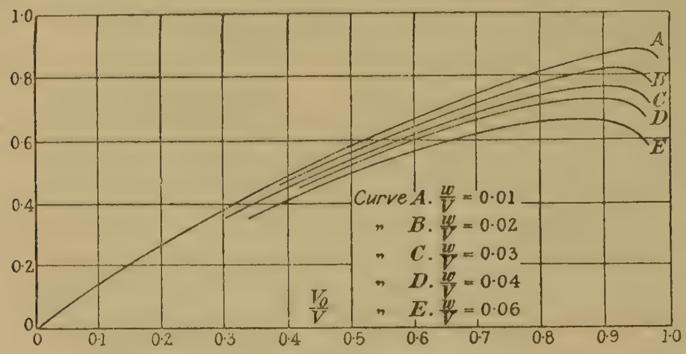


FIG. 7.

A working life of perhaps 3,000 hours would, however, be expected under these conditions.

7. *The Regulation.*

Assuming that for a particular adjustment of the rectifier I_e is constant and that the transformer voltage remains constant, then the "regulation" may be taken to describe the variation of V_0 with I_0.

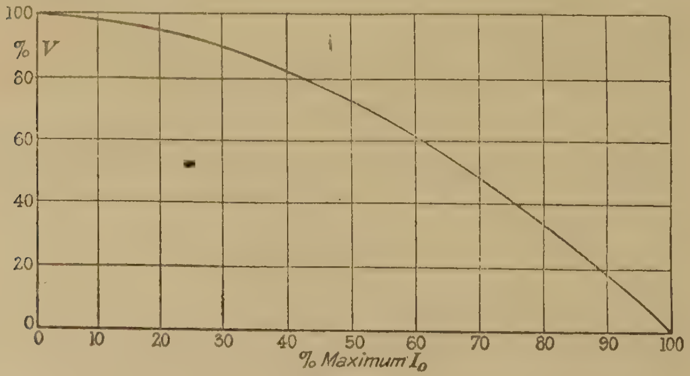


FIG. 8.

This can be determined from the curves of Fig. 4. For every value of the ratio V_0/V the corresponding value of

I_e/I_0 can be found. Thence, giving V and I_e particular values, the corresponding values of V_0 and I_0 can be ascertained.

The maximum value of I_0 is $0{\cdot}5I_e$, and the maximum value of V_0 is V. In Fig. 8 the regulation has been plotted as a curve of the percentage of the maximum value of V_0, viz., V, against the percentage of the maximum value of I_0, viz., $0{\cdot}5I_e$.

8. *The Corrections for the Various Assumptions that have been made.*

(*a*) The assumptions with respect to the periods *BC* and *DE* of Fig. 4.

The assumptions made in neglecting these periods are incorrect in that :—

(i.) The voltage V_0 varies below and above the average value during the conductive period.

(ii.) The electron current rises gradually during the time that the excess plate voltage is less than the voltage required to produce the full saturation electron current.

The effect of the variation of V_0 is that in the early part of the conductive period the electron current starts earlier than is assumed ; and in the latter part of the conductive period the fall of the electron current begins earlier than is assumed. The two effects, therefore, tend to neutralise one another. But since the curvature of the sine wave increases towards the apex of the wave the advance (in time) at which the fall of the current sets in is greater than the advance in time of the point at which the rise of the current begins. The result is therefore that the quantity passing through the rectifier is somewhat over-estimated.

For any given ratio of V_0/V and for any assigned value of a, the angles corresponding to the start of the current and to the fall can be found by reference to a table of sines and circular measure of angles. The error in the estimate of the angle for which the full current is passing can be found and compared with the assumed angle of $\pi - 2\theta$. Curves of percentage error for each value of V_0/V can then be plotted. This has been done in Fig. 9. It will be seen that the errors are small except where V_0/V approaches unity, and when $a/2$ is comparable to $1 - V_0/V$.

For example, if V=10,000, V_0=7,000 and a=0·2, the error is only just over 1 per cent.

The error arising from the second assumption is very much more serious. The electron current does not rise instantaneously to its full value. For any particular case the result of this supposition can be found by actually plotting the current

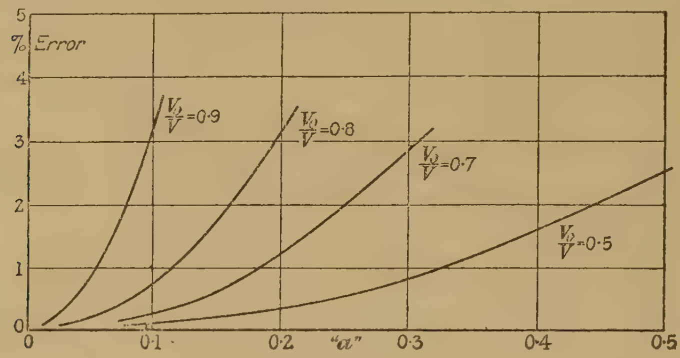


FIG. 9.

curve for the interval of time elapsing before the full current is attained. In Fig. 10 a somewhat extreme case is taken in which $V=5{,}000$, $V_0=4{,}000$, and $V'=500$. The error in the estimation of the quantity is found by taking the area beneath the actual curve and comparing it with the area assumed in

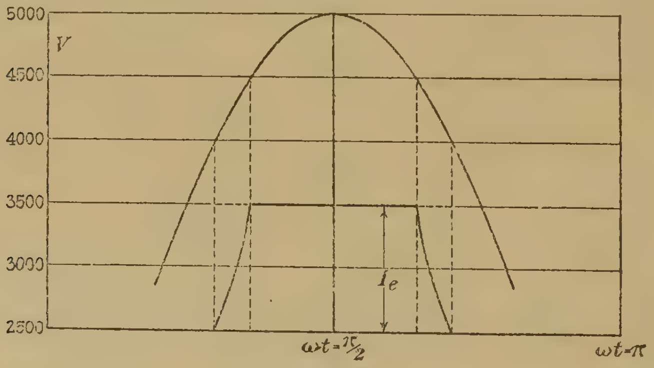


FIG. 10.

obtaining the formulæ. In this case the formulæ give an excess value of about 17 per cent.

Obviously the smaller V' becomes the smaller will be the error. For example, if $V=10{,}000$, $V_0=7{,}000$, and $V'=200$, the error is reduced to something of the order of 2 per cent.

The error introduced in this way can be estimated with a fair degree of accuracy, as follows :—

So long as V' is not very large, the value of $V-V_0\,(=v')$ may be taken as varying directly as the time, and may be written $v'=k't$, where $k'=\left(\frac{dv}{dt}\right)_{\varpi t=\theta}$.

Then at every instant the electron current $(=i')$ is $i'=Av'^{\frac{3}{2}}=A't^{\frac{3}{2}}$; A and A' being constants.

The quantity passing before the saturation value is reached is, therefore,
$$\int_0^{t'} i'dt=\frac{2}{5}A't'^{\frac{5}{2}}=\frac{2}{5}I_e t',$$
where
$$t'=\frac{1}{\varpi}\left(\sin^{-1}\frac{V_0+V'}{V}-\sin^{-1}\frac{V_0}{V}\right).$$

When deriving the formulæ it was assumed that the full current I_e was flowing for $(\pi-2\theta)/\varpi$ secs. per cycle.

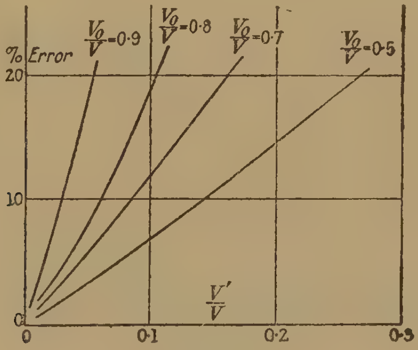


FIG. 11.

But during the initial and final periods the quantity is only $\frac{2}{5}I_e t'$, instead of $I_e t'$ as assumed. The over-estimate is therefore $\frac{6}{5}I_e t'$, which, expressed as a percentage error, is
$$120\frac{\varpi t'}{\pi-2\theta}=\frac{120}{\pi-\sin^{-1}V_0/V}\left(\sin^{-1}\frac{V_0+V'}{V}-\sin^{-1}\frac{V_0}{V}\right).$$

For various values of V_0/V, a series of values of V'/V can be taken, and the curves can be plotted as in Fig. 11.

From these curves it will be observed that the greatest errors correspond to the highest values of V_0/V.

For any assigned values of V and V' both these percentage corrections may be applied to any desired point on the curve of I_e/I_0 of Fig. 4.

The root-mean-square value of the current taken from the transformer is altered by these corrections in very approximately the same proportion as the current I_0.

(*b*) The error arising from neglecting the inductance and resistance of the transformer.

It has been assumed that the voltage applied to the rectifier circuit is a sinusoidal voltage $v=V \sin wt$. In practice the supply may be from alternating current mains or from an alternator. In either case the inductance and resistance of the leads, alternator and transformer can be converted into equivalent inductance and resistance in the high voltage side, in the usual way. The conditions are then represented by a sinusoidal E.M.F. applied to a rectifier circuit similar to that of Fig. 1, but with this equivalent resistance and inductance in series.

Both the inductance and the resistance will have some effect on the voltage-current characteristic of the rectifier.

So long as the inductance is not very large the resulting effect is relatively small. Regarding the current through the rectifier as an impulse, it follows that the initial increase from zero to the saturation value I_e will be delayed. But when the end of the conductive period is reached the falling-off of the current will be similarly delayed. The exact behaviour in any particular requires rather laborious numerical work, but to a fairly near approximation it may be assumed that the two effects neutralise one another, so long as the voltage across the inductance during either the BC or DE periods of Fig. 2 is not much greater than the voltage required to drag away the saturation electron current.

The resistance of the circuit has, however, a cumulative effect, in that it delays the rise of the current and advances the fall. Again, it is difficult to derive a formula to meet all cases, and results are most easily obtained by graphical methods. The rising and falling parts of the curves can be plotted on the assumption of no resistance, as in Fig. 10. For each value of the current in these curves the IR voltage drop can be calculated, and from the voltage wave the instant at which the excess voltage is reached can be found. By this means a corrected curve of the rise of I_e can be plotted, and the error estimated by taking the areas. The curve of

Fig. 12 corresponds to the curve of Fig. 10, but a correction for a resistance of 500 ohms has been taken, the saturation current being 0·5 amperes, and the potential difference required to drag this current away being 500 volts. It will be observed that the ratio of the errors due to the voltage drop in the resistance and to V' are roughly proportional to the values I_eR and V'. Thus the curves of Fig. 11 can be used as a close approximation for the correction for the resistance drop, by simply adding on to V' an amount equal to I_eR.

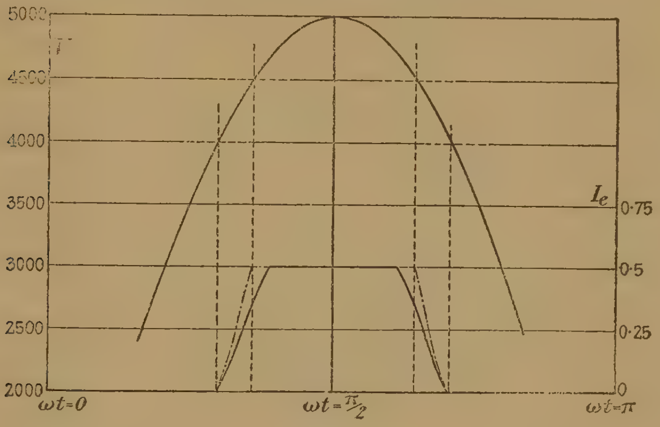


FIG. 12.

9. *Numerical Example.*

A steady current supply of 0·1 ampere at 6,000 volts is required from a 220-volt 50-cycle alternating current supply, a probable working life of the rectifiers of not less than 2,000 hours being provided for and the steady voltage variation being reduced to 1 per cent.

A probable working life of 2,000 hours for the rectifiers means with tubes of present design, that the watts per ampere of electron emission will be at least 100, and probably more. To provide some factor of safety 150 watts per ampere may be taken.

Inspection of the curves of Fig. 7 shows that a ratio of V_0/V for maximum efficiency will be in the neighbourhood of 0·9. Thus, if $V_0=6{,}000$, V will be approximately 6,700 and $W/V=150/6{,}700=0{\cdot}0225$, for which the maximum overall efficiency will occur very near to the point at which $V_0/V=0{\cdot}9$.

If the voltage ripple is not to exceed 1 per cent. with this low-frequency supply, a bi-phase circuit will certainly be used.

The transformer voltage step-up will therefore be from 220 to 9,400 volts root-mean-square. Some allowance must be made for the resistance of the windings, and a step-up of 220/10,000 would probably be specified so as to be a little on the safe side.

Referring to Fig. 4 the ratio I_e/I_0 for $V_0/V=0{\cdot}9$ is about 6·8.

In determining I_0, some allowance must be made for the corrections given in Fig. 11. As a first approximation these may be taken as 20 per cent. Thus, a total I_0 of 0·12 ampere should be calculated for.

Using the bi-phase arrangement an electron emission of $0{\cdot}12\times 6{\cdot}8/2=0{\cdot}41$ amperes would have to be provided for.

The energy to be dissipated in each rectifier is found from Fig. 4 by scaling off the value of W_R/I_0V corresponding to $V_0/V=0{\cdot}9$.

With the bi-phase arrangement the maximum voltage per tube is 7,070, and the effective value of I_0 is 0·12/2. Hence, $W_R=0{\cdot}06\times 7{,}070\times 0{\cdot}16=68$ watts.

The root-mean-square value of the current in the transformer windings is found from Fig. 5. The ratio of J/I_0, corresponding to $V_0/V=0{\cdot}9$ is 2·7. This is for a single-phase rectifier.

With the bi-phase arrangement the value of I_0 to be taken for one tube is one-half of the total.

Hence, $=2{\cdot}7\times 0{\cdot}12/2=0{\cdot}162$ ampere.

The corrections must now be considered. The voltage required for the saturation current of a tube of this output will be given by the manufacturers, and will probably be about 150 to 200 volts. Assuming $V'=200$, $V'/V=200/7{,}070=0{\cdot}3$ about.

Hence, the value of I_0 will be over estimated by about 12 per cent. The other corrections are relatively small, and need not be considered unless a very high degree of accuracy is required.

The proposed design, therefore, appears to be a little on the liberal side all through.

Whether a recalculation to meet the conditions more exactly should be carried out depends upon the conditions. If they are exactly those required this design would stand. If they in themselves involve a factor of safety, then a recalculation would be made.

It now remains to determine the capacity of the smoothing condenser.

It is specified that the ripple shall not exceed 1 per cent.—*i.e.*, $a=0{\cdot}01$.

The capacity required is given by

$$C=\frac{I_0}{V_0}\cdot\frac{1}{af}\cdot\frac{\theta}{\pi},$$

where $$\theta=\sin^{-1}\frac{V_0}{V}=\sin^{-1}0{\cdot}9=1{\cdot}12.$$

$$\therefore \qquad C=\frac{0{\cdot}1}{6{,}000}\cdot\frac{1}{0{\cdot}01\times50}\cdot\frac{1{\cdot}12}{\pi}=12\times10^{-6},$$

i.e., 12 mfd.

The outline specification of the apparatus required is thus :—

Rectifiers.—Two in number required, designed to give an electron emission of 0·41 ampere with an effective life of 2,000 hours when operating on a circuit of 5,000 volts root-mean-square. The arrangement of the leads through the glass and supports for the electrodes must be such that a voltage of 13,000 volts, with the filament positive, will be safely withstood. The plate to safely withstand an energy dissipation of 68 watts continuously without appreciable evolution of occluded gas.

Transformer.—Wound for a step-up of from 220 to 10,000 volts, the windings being designed for a root-mean-square secondary current of 0·162 ampere. Three secondary terminals are required, one at each end of the winding and one at the mid-point.

Condenser.—To be of capacity 12 mfd., and to be insulated for 6,000 volts.

Summary of Contents.

The Paper refers to the use of high-voltage thermionic tubes for the production of high-voltage steady current from an alternating-current supply. The subject is approached from the designer's standpoint. Curves and approximate methods of calculating them, are given from which the best combination of electron current and alternating supply voltage can be determined for any prescribed conditions.

Further, the curves allow of the essentials of the specification of the rectifiers and transformer being ascertained, and a formula is given for the capacity of the smoothing condenser

required to maintain the steady supply voltage within any assigned limits. Finally, correction curves are added by means of which more exact results can be obtained from the approximate curves given in the first instance.

DISCUSSION.

General G. O. Squier thought the Paper was of great interest and provided an efficient means of obtaining high-voltage direct current. He was glad to see the large number of Papers to the Physical Society dealing with the thermionic tube. The development of this subject during the war had been very rapid—at least equal to 10 years' ordinary progress. In the United States over a million per annum of these valves are now being turned out. This large output had forced the standardisation of sockets and other parts, which in itself was a very useful result. The increasing use of the valve was evident every day, and it was difficult to foresee how far it would ultimately displace existing apparatus. It was so simple and adaptable in use that it would play a leading part in the future development of physics. He knew of no other case in which there was a smaller time-lag between laboratory investigations and everyday use than in the case of the thermionic tube.

Prof. Bragg said the Paper dealt with a problem which was of great interest to physicists. For X-ray work a steady direct current was of the greatest importance, especially for those workers who wished to be able to obtain electrons moving in a tube with a definite velocity. For this work a constancy of about ½ per cent. was required. He believed this had been accomplished in America with quite large powers

Captain Tucker said he had used valves in conjunction with microphones, and hoped to develop this work so as to render the microphone more effective.

Dr. D. Owen said he had found an examination of the numerical example at the end of the Paper most enlightening. It helped one to realise that the tube was only controlling the energy which was supplied by the generator, only a small proportion being used up in the tube. Could Prof. Fortescue tell us the exact amount in the present case ?

Mr. Gossling also commented on the simplicity of the physical phenomena in thermionic valves. When the valve was working properly a stream of electrons passed from the hot filament to the anode, subject to no resistance other than their own repulsions. If there was gas enough in the tube to produce appreciable departure from this simple condition the valves gave trouble. The pressure in a valve was of the order 10^{-4} mm., and the mean free path of an electron at such a pressure was about 10^4 cm., so there was little chance of a collision between an electron and a gas molecule. The rectification by the valves was of a very high order, the reverse current under normal circumstances being about a ten-thousandth of the direct current. If the anode be raised to a high temperature and more gas than usual be present, the fraction may rise to about one in a thousand. In connection with the life of a filament, he thought one of the figures given by the author viz., 2,000 hours at 100 watts per ampere—was rather a rosy estimate. The second figure given was probably nearer the case.

Mr. Rayner said that in many cases where there was some voltage in excess of that required, an effective method of smoothing out "ripples" in the direct current was to replace the condenser C (Fig. 1) by a number of smaller condensers in parallel, separated by resistances.

Mr. Coursey stated that choking coils had often been used in place of the resistances suggested by Mr. Rayner, and had been found effective. What was the basis of the statement that the dimensions of the condenser were proportional to CV ? He would have thought they were proportional to CV^2.

Prof. FORTESCUE, in reply, said he had scarcely indicated in the Paper the extent to which we were indebted to Langmuir and other American workers in connection with the development of valves. At least 50 per cent. of our knowledge came from America. In reply to Dr. Owen, the efficiency was about 60 per cent. under the conditions of the demonstration. Mr. Rayner's suggestion was excellent, and had already been widely applied, as had also the use of choking coils. The beauty of the scheme was that if the first resistance and capacity reduced the alternating component of the current in the ratio $1/a$, the next reduced it to $1/a^2$, &c. Mr. Coursey's criticism was correct. The product CV_0^2 was proportional to the overall dimensions, and not the product CV_0, as stated in the proofs that had been circulated.

XXIV. *On the Measurement of Small Susceptibilities by a Portable Instrument.* By *Prof.* ERNEST WILSON.

RECEIVED MAY, 20, 1919.

IN a Paper on "The Magnetic Balance of MM. P. Curie and C. Chéneveau," * M. Chéneveau describes an instrument which involves the same fundamental principles as the portable instrument which is the subject of the present communication. It is capable of dealing with the measurement of very small susceptibilities, and its indications are recorded on a scale after the manner of an ordinary reflecting galvanometer. It employs a permanent magnet of ring form, which gives a constant intensity to the magnetic field.

In connection with some recent work on rock specimens† the author had occasion to develop an instrument for the measurement of susceptibilities of low order, which was described in a Paper read before the Royal Society on January 29, 1919. This instrument differs from that of MM. Curie and Chéneveau in that an electro-magnet is employed instead of a permanent magnet, and thus the magnetic field can be varied in intensity. It appeared desirable to construct a more simple instrument, which would give the desired sensitivity and at the same time should be portable and capable of easy and rapid adjustment.

For magnetic survey work the important range of susceptibilities is from about 0·000002, as in the case of limestones and dolomites, for example, to 0·15, as in the case of low-grade magnetities, the superior limit being about 3 or 4 in the case of magnetite crystals. The instrument to be described is capable of measuring from about 0·15 to 0·0003 with a strong suspension, and so low as 0·000015 with a weaker but sufficiently strong suspension, and indications of lower order than 0·000015 can be observed. Other illustrative cases of rock specimens are certain dolorites, 0·0045 ; ordinary grey granite, 0·001 to 0·002 ; red hæmatite, 0·0002 ; ironstone, 0·0003 ; and ruby mica in the direction of the laminations, 0·000012.

The principle underlying the action of the instrument is given by Maxwell in the equation

$$F=\tfrac{1}{2}K\frac{dH^2}{dx},$$

where F is the mechanical force per unit volume acting upon

* "Proc." Phys. Soc. Lond., 1910, Vol. XXII., Pt. III., p. 343.

† "Phil. Trans." R.S. A, Vol. 219 (Appendix), 1919.

the substance, whose susceptibility is K, and dH^2/dx is the gradient of the square of the magnetic force varying with x the distance. The specimen is preferably in the form of a short cylinder, or when in the form of powder or solution is contained in a small glass tube, and is fixed to one end of a horizontal beam, which is supported by a phosphor-bronze strip. Normally it hangs between two fixed poles of wrought iron, which are capable of being magnetised to varying degrees of strength by a permanent magnet. The torsion-head of the instrument which supports the suspended system is turned until the angle of twist between it and the beam is a maximum. Let this angle be θ, and let the volume of the specimen be V,

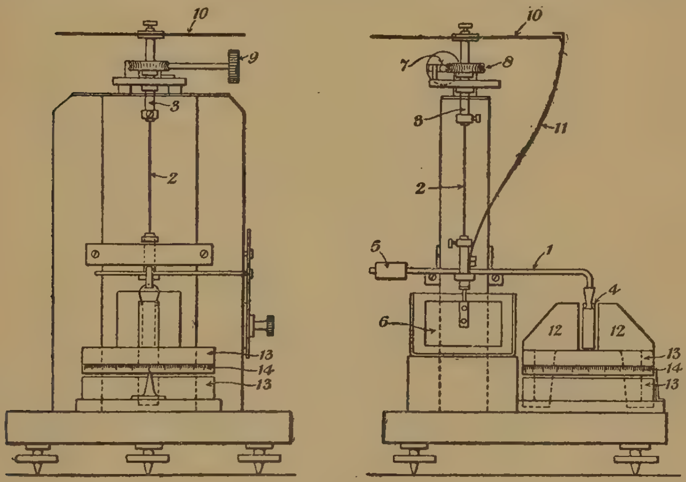


FIG. 1.

then the susceptibility K is calculable from the equation $K=C\theta/H^2V$, where H is the intensity of the magnetic field and C is a constant of the instrument.

The Instrument.

The instrument is shown in front and side elevation in Fig. 1. The horizontal beam, 1, is supported by a phosphor-bronze strip, 2, attached to the torsion-head, 3. The specimen 4 is held in a grip at one end of the beam 1, and is counterweighted by a sliding weight, 5. A vane, 6, submerged in oil serves to damp out the oscillations of the moving system, and

thus enables readings to be quickly made. Although more convenient, magnetic damping was not employed, as it was thought inadvisable to have any possibility of disturbance when small magnetic fields were used. By means of a worm, 7, and worm-wheel, 8, the torsion-head 3 can be gradually turned in either direction, a milled head, a, being provided on an extension of the worm-wheel spindle. The torsion-head carries a disc, 10, which is provided with a scale divided into 360 equal parts, and the pointer 11 attached to the moving system indicates the angular movement of the torsion-head from its normal or zero position. The specimen 4 hangs between the poles 12, 12, which are fixed to the base of the instrument. The magnet consists of two rings of tungsten steel, 13, 13, capable of rotation in a horizontal plane about an axis coinciding with the axis of the specimen, 4, when hanging symmetrically between the pole-pieces. Each ring is magnetised with north and south poles on opposite ends of a diameter, and the upper ring is provided with a scale, 14, in order that the relative position of the poles on the magnet rings can be determined. If the magnet rings are so placed that their like poles are together and coincide with the poles 12, 12, one of these becomes a north and the other a south pole, and in this case the intensity of the magnetic force between the poles is a maximum.

Two methods for variation of the force H are now available (*a*) keeping the like poles together by means of the locking device, the two rings can be turned through known angles and the force passes through zero value when the angular movement is 90 deg. from the maximum; (*b*) keeping the lower magnet ring fixed in the position of maximum force and turning the upper ring from its position of maximum force, the value of H can be reduced until the angle turned through is 180 deg., when the difference between the two magnets is obtained. In practice it is difficult to obtain an exact equality in the strength of the two rings, and, moreover, the difference between two large and nearly equal forces is liable to be seriously affected by a slight variation in one of them. During the ageing of the magnets, therefore, this difference is liable to vary. For this reason it is desirable to vary the magnetic force by turning the two rings simultaneously, thus sacrificing the more open scale available when one ring only is rotated. The second method is used for demagnetising purposes to be described later.

The Magnet Rings.

Each ring has internal and external diameters of $3\frac{1}{2}$ in. and $4\frac{1}{4}$ in. (8·9 cm. and 10·8 cm.) respectively, and its depth is $\frac{3}{4}$ in. (1·9 cm.). The material is of hardened tungsten steel, and in this connection the author is indebted to Mr. W. Carter, Messrs. J. J. Saville (Ltd.), Sheffield, for supplying these rings and for the interest he took in their preparation. A yoke piece of wrought iron having a cross-section $1\frac{1}{2}$ in. by $\frac{1}{2}$ in. (3·8 cm. by 1·27 cm.) was wound with a magnetising coil of 225 turns and bridged across a diameter of the two rings when they were finally being magnetised, so as to produce opposite poles. The magnetising force due to this yoke was reinforced by windings of 124 turns on either side of the rings.

With the yoke removed the two windings on the rings were placed in series and used as a primary, the total turns being 248, for the purpose of ballistic galvanometer tests, as it appeared desirable to find the magnetic properties of the rings. Secondary windings were placed on the rings and yoke piece. The results obtained are summarised in the following table, and are illustrated in Fig. 2. They indicate that this steel is of the highest grade for the purpose of permanent magnets :—

$H_{max.}$	$B_{max.}$	Coercive force H_0.	Retained magnetic induction B_0.	Hysteresis. Ergs per cycle per cubic cm.
152	13,700	56	10,300	213,000
95	11,800	52	8,370	151,000
74	7,430	42	4,570	70,100
36	1,770	5	250	1,990
20	840	2	62	398

The above figures were obtained from experiments with the first two rings. A second pair of rings was also available, but these were not tested for hysteresis loss. A current of 15·2 amperes was used for the highest force in the above tests, and with the yoke in position the current was put up to 20 amperes and gradually removed. In the case of the second set of rings the final current with yoke in position was 30 amperes.

The ageing of these rings is one of the vital points in the instrument. It was determined by measuring the force between the poles of the magnet. For this purpose a coil having the same dimensions as the poles was attached to a ballistic galvanometer, and deflections were obtained on quickly

removing it from the gap. After much experimenting the first set of rings has aged 31·5 per cent. from March 14th to May 19th, and the second set has aged 39 per cent. It remains to be seen if this particular method of producing a variable field is successful from the standpoint of ageing. Experiments have been made with other arrangements of magnets and pole-pieces and these are being further investigated.

Demagnetisation.

When it is desired to demagnetise the pole-piece or a specimen after the application of a large force the rings were locked relatively to one another at intervals gradually increasing the angular displacement between the poles, and thus diminishing the maximum field. At each interval the two rings, so locked together, were rotated a few times. A series of curves was obtained showing how the force varied during each operation. The curves show a gradually diminishing amplitude combined with variable phase displacement.

The Pole-pieces.

From the standpoint of convenience in working with the instrument the type of poles shown in Fig. 1 is satisfactory, but experiments with various types show that the maximum magnetic force for a given pair of rings and given gap between the pole-pieces can be increased by using two poles which are in the plane of the rings and not elevated as shown. The gain is considerable, and amounts to about 30 per cent., the difference being caused by leakage. For this reason the latter type of poles has been adopted. They are cut from a bar of good wrought iron $1\frac{1}{2}$ in. by $1\frac{1}{2}$ in. (3·8 cm. by 3·8 cm.) in section. The width where they come in contact with the inner circumference of the rings is curved to a radius of $1\frac{3}{4}$ in. (4·4 cm.), and the fit is a good one. The width of the pole-piece itself is $\frac{1}{2}$ in. (1·27 cm.), and the corners are slightly bevelled. The gap length is 1·26 cm. Tests made with the exploring coil show that after the application of a force in the gap (H) of 327 C.G.S. units the magnetism retained when the rings are removed is $2\frac{1}{4}$ per cent., but this could be removed by the method above described.

The Phosphor-Bronze Suspension.

Two sizes of phosphor-bronze strip have been used. The strong one has a rectangular cross-section of 0·85 mm. by

0·09 mm., and its length is 10 cm. At a radius of 10·9 cm. the force in dynes per 1 deg. of twist was measured, and found to be 1·50. This strip does not take up an appreciable permanent set when one end is twisted through 180 deg. The smaller section of strip is 0·85 mm. by 0·04 mm., and experi-

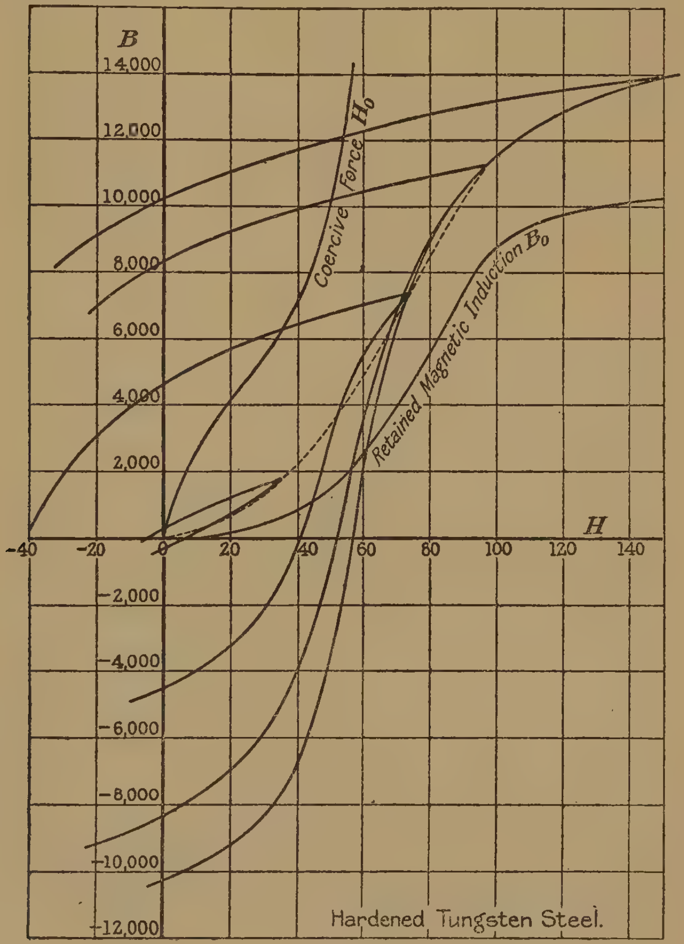


Fig. 2.

ment shows that the ratio of the restoring forces for equal lengths is 20·6 to 1 in the two cases. With this smaller section of strip the observed deflection is slightly dependent upon loading due to heavy and light specimens.

The Constant of the Instrument.

From previous experiments with the more elaborate instruments several specimens of widely differing susceptibilities were available. They are not truly circular in section, as when dealing with rock specimens much time would be spent in making them so. Table I. gives some of the figures obtained with the two suspensions with the magnet rings in the position of maximum force, and it will be seen that the ratio of the instrumental constants is about the same as that of the restoring forces in the suspensions. The considerable variation in the value of the constant is due in part to the fact that the specimens are not exactly alike as regards shape and size. Also in the case of rock specimens the susceptibility is liable to vary throughout the mass (*loc. cit.*).

Variation of H with Angular Position.

The variation of the magnetic force H with angular movement of the two magnet rings, locked together with their like poles coincident, was tested by the exploring coil and ballistic galvanometer, and also by the deflections obtained with two specimens in the gap. In Table II. values of H so obtained are compared with the values calculated on the assumption of the sine law. The agreement is such that given the maximum value of H, the force for other positions can be calculated. It is thus possible to apply known forces when making determinations, and this is important in some cases in which the susceptibility is not a constant, and is some function of H. But even when the susceptibility is constant it is necessary for a given suspension and large range to vary the force; that is to say, the phosphor-bronze strip must not be twisted beyond the elastic limits, and, therefore, when the susceptibility is large a small value of H is imperative. It will be seen that the dolerite mentioned above necessitated a twist of 160 deg., when the value of H was at a maximum. For higher susceptibilities it would be advisable to reduce the field or the volume of the specimen, the former being preferable.

The Use of Powders.

If rock specimens are ground in a non-magnetic mortar, and then tested for susceptibility, in a large number of cases the susceptibility of the original rock can be fairly closely inferred

TABLE I.

Description.	Instrumental readings. Left.	Instrumental readings. Right.	Mean value $\theta°$.	Previous determination of susceptibility.	Instrumental constant C.
Strong Suspension—					
Grey granite	91	98	94·5	0·0021	7·75
Ironstone	11	11	11·0	0·0003	9·80
Chalybite	40·5	41·5	41	0·0006	9·5
Red hæmatite	15	16	15·5	0·00022	9·5
Dolerite	156	164	160	0·0047	8·4
Weak Suspension—					
Kaolin and chalybite mixture unbaked	103	92	97·5	0·000153	0·50
Manganese sulphate, normal solution	13·5	6·5	10	0·000018	0·43

TABLE II.

Sine curve.			Ballistic test. First set of rings.		Ballistic test. Second set of rings.		Grey granite, $K=0{\cdot}0021$.			Kaolin-Chalybite, $K=0{\cdot}000153$.		
Angle.	Cosine.	$H_{max.}$ 345.	Deflection.	H.	Deflection.	H.	$\theta°$.	$\sqrt{\theta°}$.	H.	$\theta°$.	$\sqrt{\theta°}$.	H.
0	1	345	305	345	413	345	53	7·28	345	99	9·95	345
15	0·966	333	300	339	406	339	51·25	7·15	340	97·5	9·87	340
30	0·866	298	267	302	365	305	40·25	6·34	300	82	9·05	314
45	0·707	244	220	249	297	248	26·5	5·15	244	53·75	7·33	254
60	0·500	172	165	187	222	186	13·5	3·67	174	27	5·2	180
75	0·259	89·5	98	111	131	109	3·5	1·87	88·5	9·75	3·12	108
90	0	0	+30	34	+44	37	0	0	0	0	0	0
97·5	...	...	−11	12·4	−6	5	...	...	...	...	...	...

(*loc. cit.*). In magnetic survey work it is important to test a large number of specimens of the same rock in order to obtain an average. It may therefore be an advantage to grind such specimens, as the work entailed by cutting to shape would be prohibitive. When powders are used they are contained in a small glass tube which has a diameter of about 1 cm. and a length of about 4 cm., correction being made if necessary for the glass in the tube.

SUMMARY.

A horizontal beam is supported by a phosphor-bronze strip attached to a torsion head. The specimen is held in a grip at one end of the beam, and is counterpoised by a sliding weight. The specimen hangs between pole-pieces which are fixed to the base of the instrument. The magnet consists of two rings of tungsten steel, capable of rotation in a horizontal plane about an axis coinciding with the specimen. Each ring is magnetised with north and souch poles at opposite ends of a diameter. By varying the relative azimuths of the rings the field between the pole-pieces can be varied. The method of test consists of turning the torsion head until the specimen just breaks away from the field between the poles. If the torsion angle be θ and the volume of the specimen be V, the susceptibility K is given by $K=C\theta/H^2V$, where H is the intensity of the field and C is a constant.

DISCUSSION.

Mr. R. S. Whipple thought the author's method of varying the field by means of rotating ring magnets was very ingenious. In testing the parts of moving-coil instruments for traces of magnetism they had found it very valuable to take the time of swing in a magnetic field and in a zero field. He was inclined to think that this was the simplest and possibly most accurate method of measuring small susceptibilities.

Mr. J. Guild suspected that the instrument would be particularly liable to zero changes due to subpermanent sets in the suspension.

Prof. Lees asked if with different specimens the maximum torsion occurred when the specimen was at the same point of the field. Unless this was so, dH^2/dx would not be constant, as required by the theory. Arising out of this point, what was the effect of the actual size of the specimens on the results, as dH^2/dx would not have the same value for large and small specimens.

Prof. Wilson, in reply to Mr. Whipple, said that if you had two materials of the same susceptibility but of different electrical conductivities, the times of swing would, he thought, be seriously affected by the difference in the eddy currents set up in the two cases. In reply to Mr. Guild, he had found no zero change effects after torsions of 180 deg. With regard to the President's points, it was necessary in order to get consistent results to use specimens all about the one size. As regards the position in the field at which the specimen breaks away, this did vary slightly with the strength of the field; but he was not sure that this would appreciably affect the torsion, which was what the instrument actually measured.

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PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1918–1919.

October 25, 1918.

Meeting held at Imperial College of Science.

Prof. C. H. LEES, F.R.S., President, in the Chair.

A Discussion took place on "The Case for a Ring Electron," opened by Dr. H. S. ALLEN, M.A.

November 8, 1918.

Meeting held at Imperial College of Science.

Prof. C. H. LEES, F.R.S., President, in the Chair.

The following Papers were read :—

1. "Low-Voltage Arcs in Metallic Vapours." By Prof. J. C. McLENNAN, F.R.S.

2. "Relativity and Gravitation." By Dr. W. WILSON.

A Demonstration of Experiments Illustrating Colour Blindness was given by Mr. C. R. GIBSON.

November 22, 1918.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1. " Note on the Linguistic Nomenclature of Scientific Writers." By A. Campbell, B.A.

2. " A Note on Low Frequency Microphone Hummers." By A. Campbell, B.A.

3. " A Simple Tuning Fork Generator for Sine-Wave Alternating Current." By A. Campbell, B.A.

4. " A Method of Comparing Tuning Forks of Low Frequency and of Determining their Damping Decrements." By A. Campbell, B.A.

5.*" Cohesion " (Fifth Paper). By Prof. H. Chatley.

* Taken as read in absence of Author.

January 24, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Papers were read :—

1. " Notes on Lubrication." By S. Skinner, M.A.

2. " On Sir Thos. Wrightson's Theory of Hearing." By Prof. W. B. Morton, M.A.

3. " Electrical Theorems in connection with Parallel Cylindrical Conductors." By Dr. A. Russell, M.A.

Annual General Meeting.

February 14, 1919.

Mr. W. R. Cooper, M.A., in the Chair.

In opening the meeting the Chairman referred to the loss the Society had sustained by the death of Prof. G. Carey Foster, one of the Founders, and mentioned that the President, Prof. C. H. Lees, was representing the Society at the funeral that afternoon.

The Annual Report of the Council was read by Prof. Eccles.

In the year 1918 twelve ordinary Meetings were held. The average attendance at the Meetings was 35.

The fourth Guthrie Lecture was delivered at the Meeting on March 22, 1918, by Professor J. C. McLennan, F.R.S. The subject of the lecture was "The Origin of Spectra."

The Annual Exhibition of Apparatus was again suspended owing to the War.

During the past session the Council resolved to devote some of the Meetings of the Society in each session to the discussion of selected subjects of current interest. This year a discussion on "The Teaching of Physics in Schools" attracted an audience of 101 Fellows and visitors.

In the autumn of 1917 the Council appointed a Committee to examine into the possibility of improving the Professional Status of the Physicist. Acting upon the report of this Committee, a Conference was arranged with representatives from other Societies, which has resulted in the proposal to establish an Institute of Physics of a professional type.

Professor C. H. Lees and Dr. S. W. J. Smith were nominated by the Council to serve on the Conjoint Board of Scientific Societies.

During the session the Society published a Report on the "Relativity Theory of Gravitation," by Professor A. S. Eddington, M.A., M.Sc., F.R.S.

The number of Honorary Fellows on the roll on 31st December, 1918, was 10, and the number of Ordinary Fellows 446. Twenty-seven new Fellows and one Student were elected during the year, and there was one resignation.

The Society has to record with regret the deaths of Mr. E. Russell Clarke, M.B.E., Mr James Enright, B.Sc., Professor O. Henrici, F.R.S., Professor B. Hopkinson, F.R.S., Mr. Sydney Lupton, M.A., and Sir H. T. Wood, B.A.

The Report of the Treasurer was read by Mr. W. R. Cooper.

The accounts for the past year show the position of the Society to be much sounder than in 1917, there being a credit balance of £98. 9s. 8d., as against a debit balance of £48. 17s. 9d. This is particularly satisfactory having regard to the fact that the expenditure was greater by £153. 9s. 2d. The heavier expenditure is due to the greatly increased cost of printing. The cost of printing and issuing the publications and notices amounted to £464. 10s. 8d., as compared with £384. 5s 4d. in the previous year; and, in addition, the Report on Relativity cost £77. 3s. 9d. In accordance with previous accounts, the publications have been taken over the complete volume (or session), which does not coincide with the financial year.

The revenue from subscriptions improved, and a considerable sum in arrears has been paid, though not as much as was hoped. Further, a sum of £70. 12s. 3d., due on account of income tax, has been recovered in respect of the two years 1916/17 and 1917/18.

The sale of publications has fallen off to a small extent, probably due to war conditions.

The balance-sheet shows that the outstanding subscriptions are still heavy, the amount being £180. 12s. It is doubtful if anything like this sum will be realised, and therefore a reserve of £80. 12s. has been set against this figure.

The investments, which have been valued at market prices through the courtesy of the London County, Westminster & Parr's Bank, have appreciated somewhat since the last balance-sheet. A sum of £100 has been invested during the year in National War Bonds.

Both Reports were unanimously adopted.

FROM JANUARY 1ST TO DECEMBER 31ST, 1918.

INCOME.	£	s.	d.	£	s.	d.
Entrance Fees	28	7	0			
Subscriptions by Fellows..................	485	2	0			
" Voluntary	15	15	0			
" by Students	2	12	6			
" Arrears paid	78	0	6			
" Paid in Advance	11	7	6			
" for "Science Abstracts" and Advance Proofs	16	19	4			
				638	3	10
Composition Fees				63	0	0
Dividends :—						
Furness Debenture Stock	11	15	10			
Midland Railway	29	5	0			
Metropolitan Board of Works......	5	1	6			
Lancaster Corporation Stock	8	14	0			
New South Wales	6	9	0			
London, Brighton & South Coast Railway	18	19	8			
Great Eastern Railway	14	15	0			
India 3½% Stock	12	13	10			
Exchequer Bonds, 5%	20	0	0			
National War Bonds, 5%	1	14	9			
				129	8	7
Income Tax refunded				70	12	3
Interest on deposit account............				6	1	0
Sales of Publications (Fleetway Press, Ltd.)				148	2	4
				£1,055	8	0

EXPENDITURE.	£	s.	d.	£	s.	d.
"Science Abstracts"......................	277	12	0			
" " Extra copies ...	3	9	6			
				281	1	6
Fleetway Press, Ltd. :—						
"Proceedings"	345	16	4			
Bulletin	38	14	11			
Distribution (Postage)............	37	12	10			
General	42	6	7			
				464	10	8
Report on Relativity.............				77	3	9
Reporting	43	16	8			
Refreshments and Attendance.........	15	0	10			
Guthrie Lecture—Honorarium	10	0	0			
Report on Relativity—Honorarium	20	0	0			
Royal Asiatic Society..................	2	2	0			
Petty Cash—						
Secretaries' Expenses	16	16	11			
Treasurer's Expenses	4	8	8			
Expenses in regard to Institute of Physics	4	0	1			
Bank Charges		2	3			
Insurance	6	10	0			
Advertising	1	5	0			
				124	2	5
Donation—Conjoint Board of Scientific Societies............................				10	0	0
Balance, being excess of income over expenditure				98	9	8
				£1,055	8	0

W. R. COOPER, *Honorary Treasurer*.

Audited and found correct,

T. MATHER
W. A. J. O'MEARA } *Honorary Auditors*.

January 28th, 1919.

BALANCE SHEET AT DECEMBER 31ST, 1918.

ASSETS.

	£	s.	d.	£	s.	d.
Subscriptions in arrears	180	12	0			
Less reserve for subscriptions probably unrealisable	80	12	0			
				100	0	0
Investments (valued at Dec. 31) :—						
£533 Furness 3 per cent. Debenture Stock	287	0	0			
£1,600 Midland Railway 2½ per cent. Perpetual Preference Stock	744	0	0			
£200 Metropolitan Board of Works 3½ per cent. Consolidated Stock	173	0	0			
£400 Lancaster Corporation 3 per cent. Redeemable Stock	228	0	0			
£254 2s. 9d. New South Wales 3½ per cent. Ordinary Stock	23	0	0			
£500 London, Brighton & South Coast Railway Ordinary Stock...	385	0	0			
£500 Great Eastern Railway 4 per cent. Debenture Stock	380	0	0			
£500 India 3½ per cent. Stock......	347	0	0			
£400 Exchequer 5 per cent. Bonds, 1921	400	0	0			
£100 5% National War Bonds 1928	100	0	0			
				3,275	0	0
Outstanding Credit on Sales				11	2	9
Stock of Publications (Treasurer's valuation)				250	0	0
Cash at Bank on Deposit				50	0	0
Cash at Bank, Current Account at Dec. 31	101	10	1			
Adjustment for outstanding cheques	31	6	9			
				70	3	4
Cash in hand (Treasurer's Petty Cash)				1	16	8
				£3,758	2	9

W. R. COOPER, *Honorary Treasurer.*

LIABILITIES.

	£	s.	d.
Life Compositions	1,892	0	0
Balance, General Fund	1,866	2	9
	£3,758	2	9

Audited and found correct,

January 28th, 1919.

T. MATHER
W. A. J. O'MEARA } *Honorary Auditors.*

LIFE COMPOSITION FUND AT DECEMBER 31ST, 1918.

	£	s.	d.
149 **Fellows paid £10**	1,490	0	0
3 **Fellows paid £15**	45	0	0
5 **Fellows paid £21**	105	0	0
8 **Fellows paid £31. 10s.**	252	0	0
	£1,892	0	0

W. R. COOPER, *Honorary Treasurer.*

Audited and found correct,

January 28th, 1919.

T. MATHER
W. A J. O'MEARA } *Honorary Auditors.*

After the customary votes of thanks, the election of Officers and Council took place, the new Council being constituted as follows :—

President.—Prof. C. H. Lees, D.Sc., F.R.S.

Vice-Presidents, who have filled the office of President.—Prof. R. B. Clifton, M.A., F.R.S.; Prof. A. W. Reinold, C.B., M.A., F.R.S.; Sir W. de W. Abney, R.E., K.C.B., D.C.L., F.R.S.; Prin. Sir Oliver J. Lodge, D.Sc., LL.D., F.R.S.; Sir R. T. Glazebrook, C.B., D.Sc., F.R.S.; Prof. J. Perry, D.Sc., F.R.S.; C. Chree, Sc.D., LL.D., F.R.S.; Prof. H. L. Callender, M.A., LL.D., F.R.S.; Prof. A. Schuster, Ph.D., Sc.D., F.R.S.; Sir J. J. Thomson, O.M., D.Sc., F.R.S.; Prof. C. Vernon Boys, F.R.S.

Vice-Presidents.—Prof. W. Eccles, D.Sc.; Prof. J. W. Nicholson, M.A., D.Sc., F.R.S.; Prof. O. W. Richardson, M.A., D.Sc., F.R.S.; R. S. Willows, M.A., D.Sc.

Secretaries.—H. S. Allen, M.A., D.Sc.; F. E. Smith, O.B.E., F.R.S.

Foreign Secretary.—Sir R. T. Glazebrook, C.B., D.Sc., F.R.S.

Treasurer.—W. R. Cooper, M.A., B.Sc.

Librarian.—S. W. J. Smith, M.A., D.Sc., F.R.S.

Other Members of Council.—Prof. E. H. Batron, D.Sc., F.R.S.; Prof. W. H. Bragg, C.B.E., M.A., F.R.S.; C. R. Darling, F.I.C.; Prof. A. S. Eddington, M.A., M.Sc., F.R.S.; D. Owen, D.Sc.; C. E. S. Phillips, F.R.S.E.; E. H. Rayner, M.A.; S. Russ, M.A., D.Sc.; T. Smith, B.A.; F. J. W. Whipple, M.A.

After the conclusion of the general business the chair was taken by the President, Prof. C. H. Lees. The following Papers were read :—

1. "The Temperature Coefficient of Tensile Strength of Water." By S. Skinner, M.A., and R. W. Burfitt, B.Sc.

2. "Vector Diagrams of Some Oscillatory Circuits used with Thermionic Tubes." By Prof. W. H. Eccles.

3. "A Small Direct-Current Motor using Thermionic Tubes instead of Sliding Contacts." By Prof. Eccles and Mr. F. W. Jordan.

February 28, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Paper was read :—

"On Simplified Inductance Calculations, with Special Reference to Thick Coils." By Mr. P. R. Coursey, B.Sc.

A Demonstration of Some Acoustic Experiments in Connection with Whistles and Flutes was given by Dr. R. Dunstan.

A Demonstration of a New Polariser was given by Mr. G. Brodsky.

March 14, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

The following Paper was read :—

"Some Characteristics of the Spark Discharge and its Effect in Igniting Explosive Mixtures." By Messrs. C. C. Paterson and N. R. Campbell.

A Demonstration of the Uses of Invisible Light in Warfare was given by Prof. R. W. Wood.

March 28, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

A Discussion on "Metrology in the Industries" was held. Opener—Sir R. T. Glazebrook, C.B., F.R.S.

May 9, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

A Demonstration of a New Colour Transparency Process for Illustrating Scientific Lectures was given by A. E. Bawtree, F.R.P.S.

The following Papers were read :—

1. " Absolute Scales of Pressure and Temperature." By F. J. W. Whipple, M.A.

2. " On the Transmission of Speech by Light." By A. O. Rankine, D.Sc.

May 23, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. Lees, F.R.S., President, in the Chair.

An Exhibition of a Tuning Fork maintained in vibration by means of a Triode Valve was given by Prof. W. H. Eccles.

The following Papers were read :—

1. " A Form of Knudsen's Vacuum Manometer." By Mr. L. F. Richardson.

2. " On Theories of Thermal Transpiration." By Mr. G. D. West.

June 13, 1919.

Special General Meeting, held at Imperial College of Science.

It was decided that the war-time practice of holding all meetings at 5 p.m. be continued.

It was proposed by the PRESIDENT and seconded by Mr. C. R. DARLING that the Society agree to the following proposals, made by the Institute of Physics :—

" The reduction of annual subscriptions by a member of any one Society on his joining a second Society (or more) shall be as follows for each of the Societies of which he is a member :—

$33\frac{1}{3}$ per cent. in the case of members of 4 or more Societies.
25 per cent. in the case of members of 3 Societies.
15 per cent. in the case of members of 2 Societies.

" The scheme would apply to persons already members of more than one Society as well as to new members. In the event of any person already a member of one or more Societies joining another Society of the group, the entrance fee usually charged by that Society shall be waived.

" A reduction of $33\frac{1}{3}$ per cent. in the published annual price of periodical publications shall be made by the Societies to members of any of the participating Societies."

The motion was carried unanimously.

After the general business the following Papers were read :—

1. *" A Comparison of the Wave-form of the Telephone Current Produced by a Thermal Detector and by a Rectifier in Heterodyne Reception." By Dr. BALTH. VAN DER POL, Junr.

2. " The Magnetic Properties of Varieties of Magnetite." By Prof. E. WILSON and Prof. E. F. HERROUN.

* Taken as read in the absence of the Author.

June 27, 1919.

Meeting held at Imperial College of Science.

Prof. C. H. LEES, F.R.S., President, in the Chair,

The following Papers were read :—

1. " The Current Voltage Characteristics of High-Voltage Thermionic Rectifiers." By Prof. C. L. FORTESCUE.

2. "On the Measurement of Small Susceptibilities by a Portable Instrument." By Prof. E. WILSON.

July 11, 1919.

Extra Meeting held at the National Physical Laboratory, Teddington, by invitation of the Director, Sir R. T. GLAZEBROOK, C.B., F.R.S

A number of demonstrations of work in progress at the Laboratory were given.

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